5-4 Other Types of Distributions
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\[
P(X) = \frac{n!}{X_1!X_2!X_3! \ldots X_k!} \times p_1^{x_1} \times p_2^{x_2} \times p_3^{x_3} \times \ldots \times p_k^{x_k}
\]
5-4 Other Types of Distributions

The **multinomial distribution** is similar to the binomial distribution but has the advantage of allowing one to compute probabilities when there are more than two outcomes.

\[
P(X) = \frac{n!}{X_1! X_2! X_3! \ldots X_k!} \times p_1^{X_1} \times p_2^{X_2} \times p_3^{X_3} \times \ldots \times p_k^{X_k}
\]

The binomial distribution is a special case of the multinomial distribution.
Example 5-24: Leisure Activities

In a large city, 50% of the people choose a movie, 30% choose dinner and a play, and 20% choose shopping as a leisure activity. If a sample of 5 people is randomly selected, find the probability that 3 are planning to go to a movie, 1 to a play, and 1 to a shopping mall.
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\]

\[
P(X) = \frac{5!}{3!1!1!} \times (0.50)^3 (0.30)^1 (0.20)^1
\]
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\]

\[
P(X) = \frac{5!}{3!1!1!} \times (0.50)^3 \times (0.30)^1 \times (0.20)^1 = 0.15
\]
Other Types of Distributions
Other Types of Distributions

- The **Poisson distribution** is a distribution useful when \( n \) is large and \( p \) is small and when the independent variables occur over a period of time.

- The Poisson distribution can also be used when a density of items is distributed over a given area or volume, such as the number of plants growing per acre or the number of defects in a given length of videotape.
Other Types of Distributions

Poisson Distribution

The probability of $X$ occurrences in an interval of time, volume, area, etc., for a variable, where $\lambda$ (Greek letter lambda) is the mean number of occurrences per unit (time, volume, area, etc.), is

$$P(X; \lambda) = \frac{e^{-\lambda} \lambda^X}{X!}$$

where $X = 0, 1, 2, \ldots$.

The letter $e$ is a constant approximately equal to 2.7183.
Chapter 5
Discrete Probability Distributions

Section 5-4
Example 5-27
Page #285
Example 5-27: Typographical Errors

If there are 200 typographical errors randomly distributed in a 500-page manuscript, find the probability that a given page contains exactly 3 errors.
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$$P(X; \lambda) = \frac{e^{-\lambda} \lambda^X}{X!} = \frac{e^{-0.4} (0.4)^3}{3!} = 0.0072$$

Thus, there is less than 1% chance that any given page will contain exactly 3 errors.
Other Types of Distributions

- The **hypergeometric distribution** is a distribution of a variable that has two outcomes when sampling is done without replacement.
Other Types of Distributions

Hypergeometric Distribution

Given a population with only two types of objects (females and males, defective and nondefective, successes and failures, etc.), such that there are \( a \) items of one kind and \( b \) items of another kind and \( a + b \) equals the total population, the probability \( P(X) \) of selecting without replacement a sample of size \( n \) with \( X \) items of type \( a \) and \( n - X \) items of type \( b \) is

\[
P(X) = \frac{a \binom{X}{a} \times b \binom{n-X}{b}}{\binom{a+b}{n}}
\]

The letter \( e \) is a constant approximately equal to 2.7183.

Bluman, Chapter 5
Example 5-31: House Insurance

A recent study found that 2 out of every 10 houses in a neighborhood have no insurance. If 5 houses are selected from 10 houses, find the probability that exactly 1 will be uninsured.
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\[ a = 2, a + b = 10 \quad \Box \quad b = 8 \]
Example 5-31: House Insurance

A recent study found that 2 out of every 10 houses in a neighborhood have no insurance. If 5 houses are selected from 10 houses, find the probability that exactly 1 will be uninsured.

\[ a = 2, a + b = 10 \quad \text{and} \quad b = 8, \quad X = 1, n = 5 \rightarrow n - X = 4 \]
Example 5-31: House Insurance

A recent study found that 2 out of every 10 houses in a neighborhood have no insurance. If 5 houses are selected from 10 houses, find the probability that exactly 1 will be uninsured.

\[a = 2, \ a + b = 10 \quad \circledast \quad b = 8, \quad X = 1, \ n = 5 \rightarrow n - X = 4\]

\[
P(X) = \frac{a \cdot C_X \times b \cdot C_{n-X}}{a+b \cdot C_n}
\]
Example 5-31: House Insurance

A recent study found that 2 out of every 10 houses in a neighborhood have no insurance. If 5 houses are selected from 10 houses, find the probability that exactly 1 will be uninsured.

\[ a = 2, a + b = 10 \implies b = 8, \quad X = 1, n = 5 \implies n - X = 4 \]

\[
P(X) = \frac{a \binom{X}{n} \times b \binom{n-X}{n}}{a+b \binom{n}{n}}
\]

\[
P(X) = \frac{2 \binom{1}{1} \times 8 \binom{4}{4}}{10 \binom{5}{5}}
\]
Example 5-31: House Insurance

A recent study found that 2 out of every 10 houses in a neighborhood have no insurance. If 5 houses are selected from 10 houses, find the probability that exactly 1 will be uninsured.

\[ a = 2, \ a + b = 10 \ \text{\(\Box\)} \ b = 8, \quad X = 1, \ n = 5 \rightarrow n - X = 4 \]

\[
P(X) = \frac{a \ C_X \times b \ C_{n-X}}{a+b \ C_n}
\]

\[
P(X) = \frac{2 \ C_1 \times 8 \ C_4}{10 \ C_5} = \frac{2 \times 70}{252} = \frac{140}{252} = \frac{5}{9}
\]