8.5 χ^2 Test for a Variance or a Standard Deviation

The chi-square distribution is also used to test a claim about a single variance or standard deviation.

The formula for the chi-square test for a variance is

$$\chi^2 = \frac{(n-1)s^2}{\sigma^2}$$

with degrees of freedom d.f. = $n - 1$ and

n = sample size

s^2 = sample variance

σ^2 = population variance

Assumptions for the χ^2 Test for a Variance or a Standard Deviation

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1. The sample must be randomly selected from the population.

Assumptions for the χ^2 Test for a Variance or a Standard Deviation

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2. The population must be normally distributed for the variable under study.

Assumptions for the χ^2 Test for a Variance or a Standard Deviation

1. The sample must be randomly selected from the population.
2. The population must be normally distributed for the variable under study.
3. The observations must be independent of one another.

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Hypothesis Testing

Section 8-5

Example 8-21

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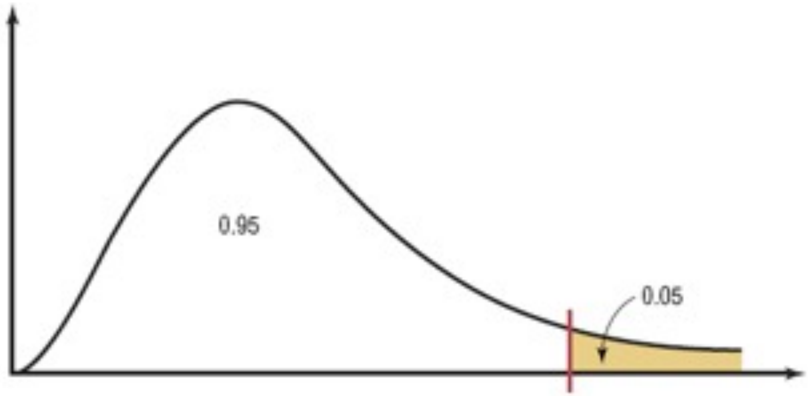
Example 8-21: Table G

Find the critical chi-square value for 15 degrees of freedom when α = 0.05 and the test is right-tailed.

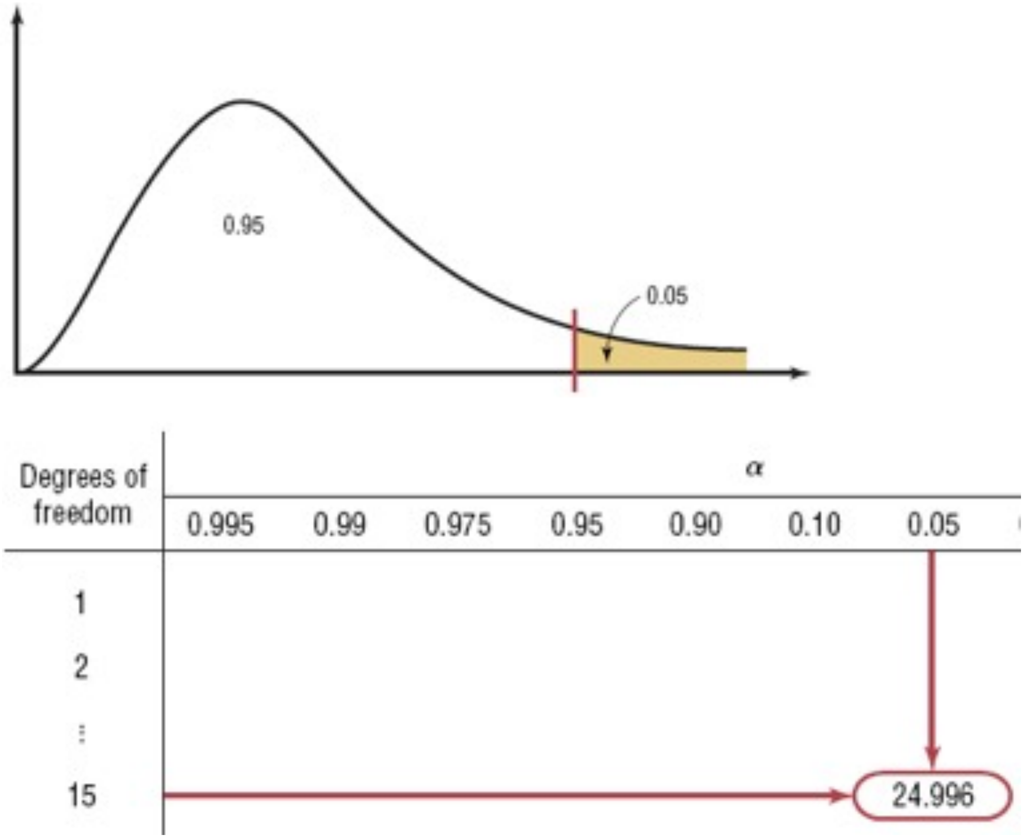
Example 8-21: Table G

Find the critical chi-square value for 15 degrees of freedom when α = 0.05 and the test is right-tailed.



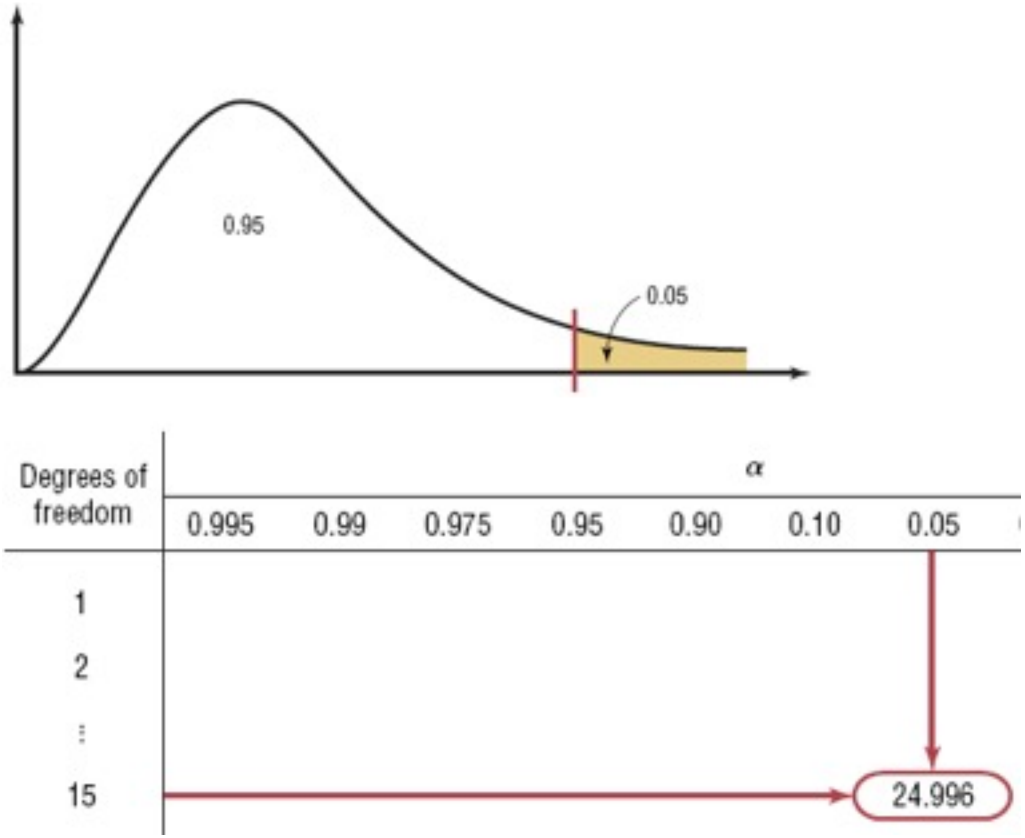
Example 8-21: Table G

Find the critical chi-square value for 15 degrees of freedom when α = 0.05 and the test is right-tailed.



Example 8-21: Table G

Find the critical chi-square value for 15 degrees of freedom when α = 0.05 and the test is right-tailed.



$$\chi^2 = 24.996$$

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Hypothesis Testing

Section 8-5

Example 8-22

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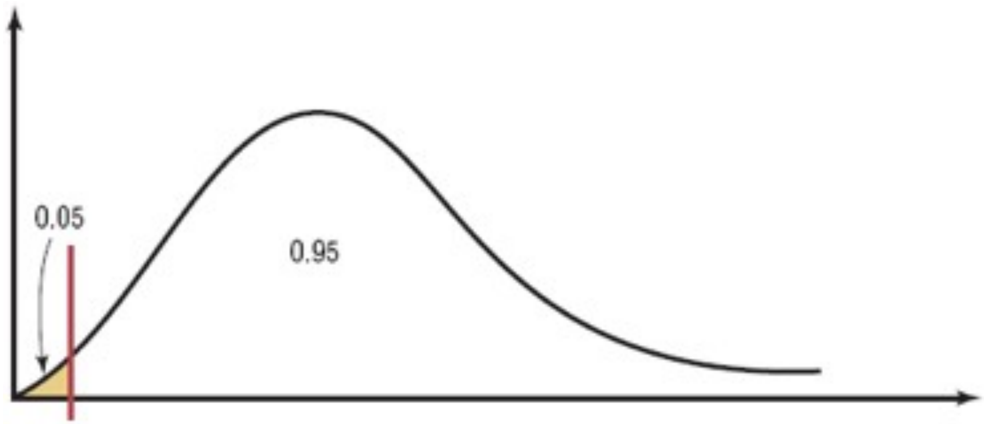
Example 8-22: Table G

Find the critical chi-square value for 10 degrees of freedom when α = 0.05 and the test is left-tailed.

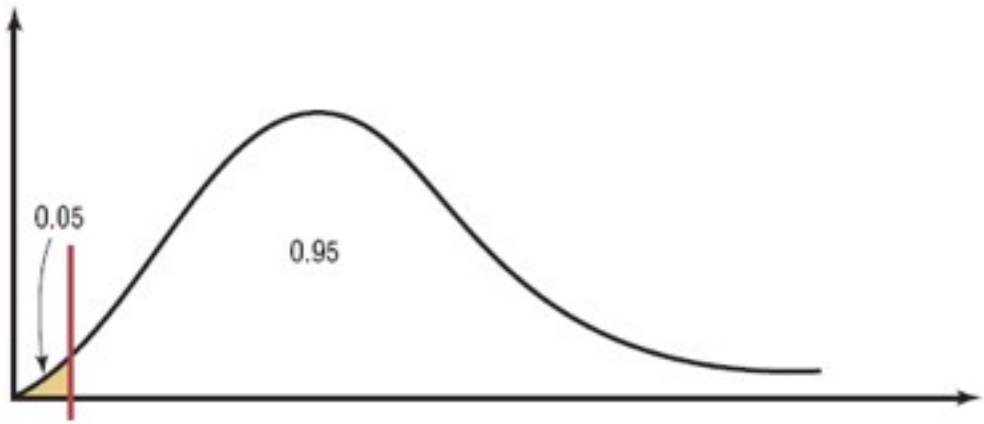
Example 8-22: Table G

Find the critical chi-square value for 10 degrees of freedom when α = 0.05 and the test is left-tailed.



Example 8-22: Table G

Find the critical chi-square value for 10 degrees of freedom when α = 0.05 and the test is left-tailed.



When the test is left-tailed, the α value must be subtracted from 1, that is, 1 – 0.05 = 0.95. The left side of the table is used, because the chi-square table gives the area to the right of the critical value, and the chi-square statistic cannot be negative.

Example 8-22: Table G

Find the critical chi-square value for 10 degrees of freedom when α = 0.05 and the test is left-tailed.

Use Table G, looking in row 10 and column 0.95.

Example 8-22: Table G

Find the critical chi-square value for 10 degrees of freedom when α = 0.05 and the test is left-tailed.

Use Table G, looking in row 10 and column 0.95.

Degrees of freedom	α									
	0.995	0.99	0.975	0.95	0.90	0.10	0.05	0.025	0.01	0.005
1										
2										
⋮										
10				3.940						
⋮										

Example 8-22: Table G

Find the critical chi-square value for 10 degrees of freedom when α = 0.05 and the test is left-tailed.

Use Table G, looking in row 10 and column 0.95.

Degrees of freedom	α									
	0.995	0.99	0.975	0.95	0.90	0.10	0.05	0.025	0.01	0.005
1										
2										
⋮										
10				3.940						
⋮										

$$\chi^2 = 3.940$$

Chapter 8
Hypothesis Testing

Section 8-5

Example 8-23

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Example 8-23: Table G

Find the critical chi-square value for 22 degrees of freedom when $\alpha = 0.05$ and a two-tailed test is conducted.

Example 8-23: Table G

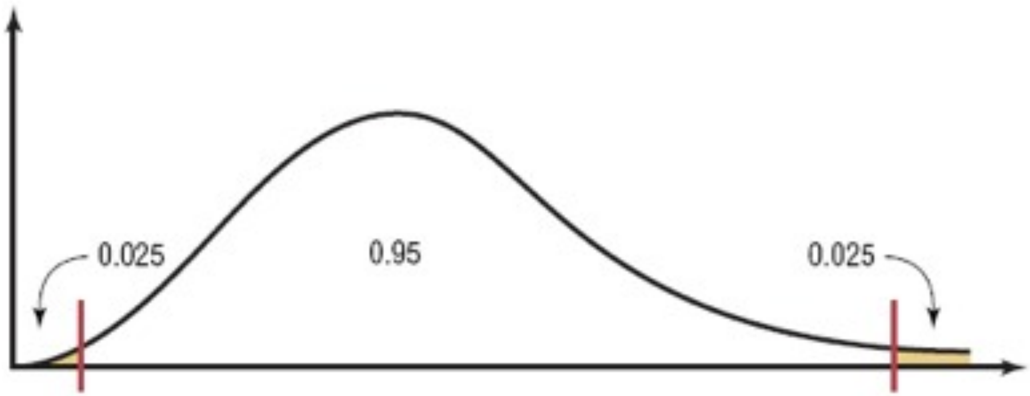
Find the critical chi-square value for 22 degrees of freedom when α = 0.05 and a two-tailed test is conducted.

When the test is two-tailed, the area must be split. The area to the right of the larger value is α /2, or 0.025. The area to the right of the smaller value is 1 – α /2, or 0.975.

Example 8-23: Table G

Find the critical chi-square value for 22 degrees of freedom when α = 0.05 and a two-tailed test is conducted.

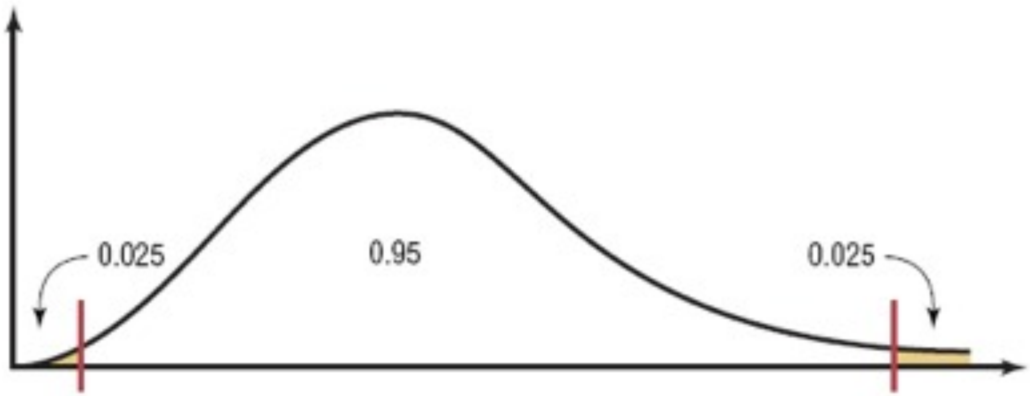
When the test is two-tailed, the area must be split. The area to the right of the larger value is α /2, or 0.025. The area to the right of the smaller value is 1 – α /2, or 0.975.



Example 8-23: Table G

Find the critical chi-square value for 22 degrees of freedom when α = 0.05 and a two-tailed test is conducted.

When the test is two-tailed, the area must be split. The area to the right of the larger value is α /2, or 0.025. The area to the right of the smaller value is 1 – α /2, or 0.975.



With 22 degrees of freedom, areas 0.025 and 0.975 correspond to chi-square values of 36.781 and 10.982.

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Hypothesis Testing

<u>Section 8-5</u>

Example 8-24

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Example 8-24: Variation of Test Scores

An instructor wishes to see whether the variation in scores of the 23 students in her class is less than the variance of the population. The variance of the class is 198. Is there enough evidence to support the claim that the variation of the students is less than the population variance (σ^2 =225) at α = 0.05? Assume that the scores are normally distributed.

Example 8-24: Variation of Test Scores

An instructor wishes to see whether the variation in scores of the 23 students in her class is less than the variance of the population. The variance of the class is 198. Is there enough evidence to support the claim that the variation of the students is less than the population variance ($\sigma^2 = 225$) at $\alpha = 0.05$? Assume that the scores are normally distributed.

Step 1: State the hypotheses and identify the claim.

H_0: $\sigma^2 = 225$ and H_1: $\sigma^2 < 225$ (claim)

Example 8-24: Variation of Test Scores

An instructor wishes to see whether the variation in scores of the 23 students in her class is less than the variance of the population. The variance of the class is 198. Is there enough evidence to support the claim that the variation of the students is less than the population variance (σ^2 =225) at α = 0.05? Assume that the scores are normally distributed.

Step 1: State the hypotheses and identify the claim.

H_0: σ^2 = *225* and H_1: σ^2 < 225 (claim)

Step 2: Find the critical value.

Example 8-24: Variation of Test Scores

An instructor wishes to see whether the variation in scores of the 23 students in her class is less than the variance of the population. The variance of the class is 198. Is there enough evidence to support the claim that the variation of the students is less than the population variance (σ^2 =225) at α = 0.05? Assume that the scores are normally distributed.

Step 1: State the hypotheses and identify the claim.

H_0: σ^2 = *225* and H_1: $\sigma^2 < 225$ (claim)

Step 2: Find the critical value.

The critical value is χ^2 = 12.338.

Example 8-24: Variation of Test Scores

An instructor wishes to see whether the variation in scores of the 23 students in her class is less than the variance of the population. The variance of the class is 198. Is there enough evidence to support the claim that the variation of the students is less than the population variance (σ^2 =225) at α = 0.05? Assume that the scores are normally distributed.

Step 3: Compute the test value.

Example 8-24: Variation of Test Scores

An instructor wishes to see whether the variation in scores of the 23 students in her class is less than the variance of the population. The variance of the class is 198. Is there enough evidence to support the claim that the variation of the students is less than the population variance (σ^2 =225) at α = 0.05? Assume that the scores are normally distributed.

Step 3: Compute the test value.

$$\chi^2 = \frac{(n-1)s^2}{\sigma^2}$$

Example 8-24: Variation of Test Scores

An instructor wishes to see whether the variation in scores of the 23 students in her class is less than the variance of the population. The variance of the class is 198. Is there enough evidence to support the claim that the variation of the students is less than the population variance (σ^2 =225) at α = 0.05? Assume that the scores are normally distributed.

Step 3: Compute the test value.

$$\chi^2 = \frac{(n-1)s^2}{\sigma^2} = \frac{(22)(198)}{225}$$

Example 8-24: Variation of Test Scores

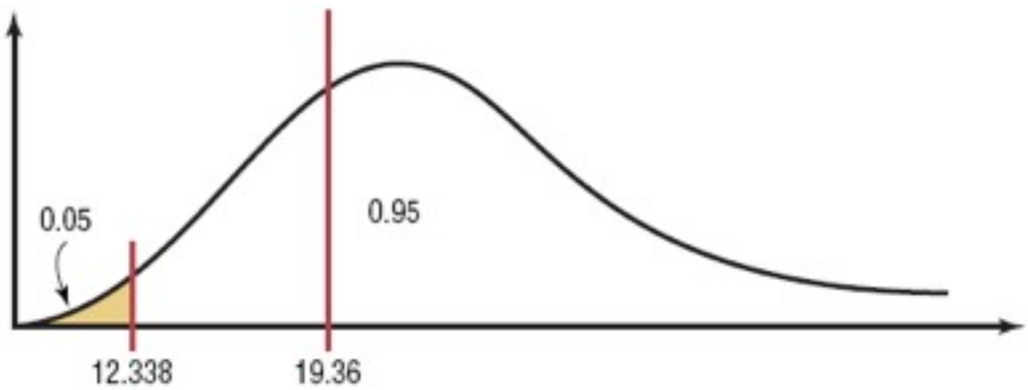
An instructor wishes to see whether the variation in scores of the 23 students in her class is less than the variance of the population. The variance of the class is 198. Is there enough evidence to support the claim that the variation of the students is less than the population variance (σ^2 =225) at α = 0.05? Assume that the scores are normally distributed.

Step 3: Compute the test value.

$$\chi^2 = \frac{(n-1)s^2}{\sigma^2} = \frac{(22)(198)}{225} = 19.36$$

Example 8-24: Variation of Test Scores

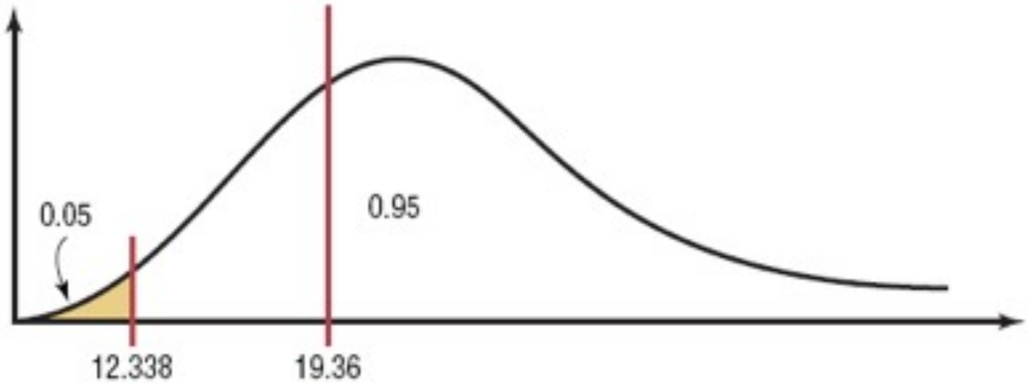
Step 4: Make the decision.



Example 8-24: Variation of Test Scores

Step 4: Make the decision.

Do not reject the null hypothesis since the test value 19.36 falls in the noncritical region.



Example 8-24: Variation of Test Scores

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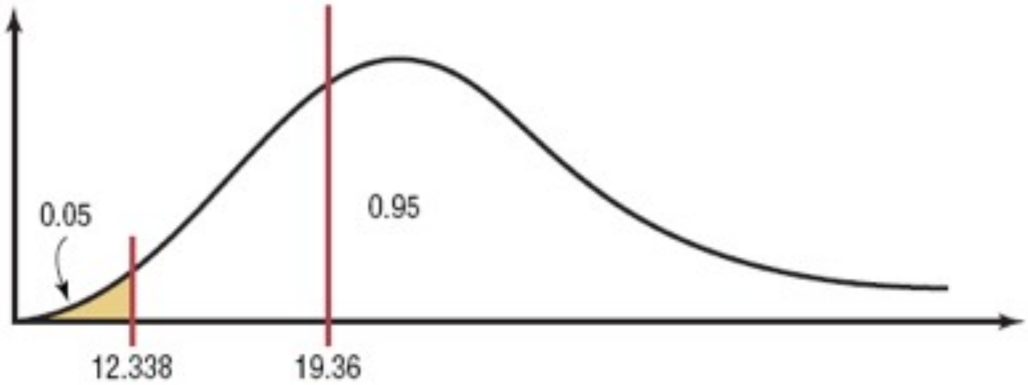

Example 8-24: Variation of Test Scores

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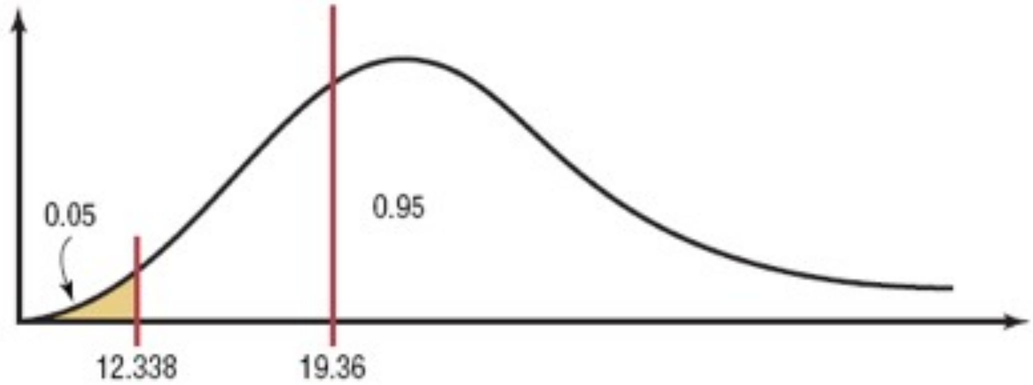

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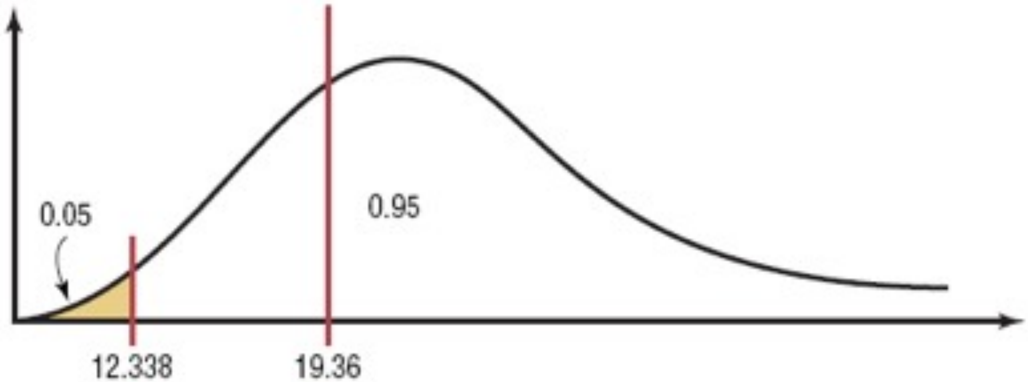

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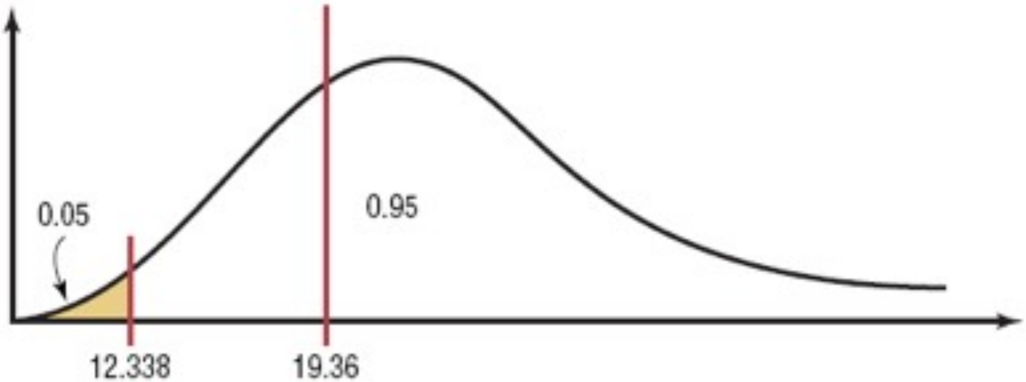
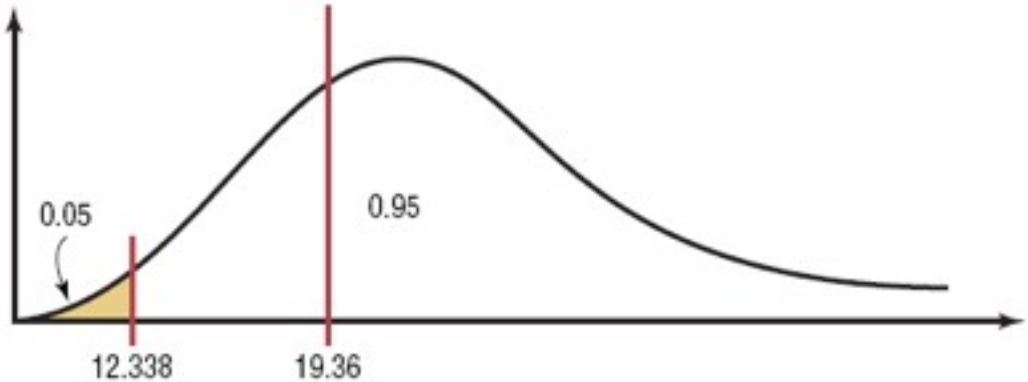


Step 5: Summarize the results.

Example 8-24: Variation of Test Scores

Step 4: Make the decision.

Do not reject the null hypothesis since the test value 19.36 falls in the noncritical region.



Step 5: Summarize the results.

There is not enough evidence to support the claim that the variation in test scores of the instructor's students is less than the variation in scores of the population.

Chapter 8
Hypothesis Testing

Section 8-5

Example 8-26

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Example 8-26: Nicotine Content

A cigarette manufacturer wishes to test the claim that the variance of the nicotine content of its cigarettes is 0.644. Nicotine content is measured in milligrams, and assume that it is normally distributed. A sample of 20 cigarettes has a standard deviation of 1.00 milligram. At $\alpha = 0.05$, is there enough evidence to reject the manufacturer's claim?

Example 8-26: Nicotine Content

A cigarette manufacturer wishes to test the claim that the variance of the nicotine content of its cigarettes is 0.644. Nicotine content is measured in milligrams, and assume that it is normally distributed. A sample of 20 cigarettes has a standard deviation of 1.00 milligram. At α = 0.05, is there enough evidence to reject the manufacturer's claim?

Step 1: State the hypotheses and identify the claim.

H_0: $\sigma^2 = 0.644$ (claim) and H_1: $\sigma^2 \neq 0.644$

Example 8-26: Nicotine Content

A cigarette manufacturer wishes to test the claim that the variance of the nicotine content of its cigarettes is 0.644. Nicotine content is measured in milligrams, and assume that it is normally distributed. A sample of 20 cigarettes has a standard deviation of 1.00 milligram. At $\alpha = 0.05$, is there enough evidence to reject the manufacturer's claim?

Step 1: State the hypotheses and identify the claim.

H_0: $\sigma^2 = 0.644$ (claim) and H_1: $\sigma^2 \neq 0.644$

Step 2: Find the critical value.

Example 8-26: Nicotine Content

A cigarette manufacturer wishes to test the claim that the variance of the nicotine content of its cigarettes is 0.644. Nicotine content is measured in milligrams, and assume that it is normally distributed. A sample of 20 cigarettes has a standard deviation of 1.00 milligram. At $\alpha = 0.05$, is there enough evidence to reject the manufacturer's claim?

Step 1: State the hypotheses and identify the claim.

H_0: $\sigma^2 = 0.644$ (claim) and H_1: $\sigma^2 \neq 0.644$

Step 2: Find the critical value.

The critical values are 32.852 and 8.907.

Example 8-26: Nicotine Content

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A cigarette manufacturer wishes to test the claim that the variance of the nicotine content of its cigarettes is 0.644. Nicotine content is measured in milligrams, and assume that it is normally distributed. A sample of 20 cigarettes has a standard deviation of 1.00 milligram. At $\alpha = 0.05$, is there enough evidence to reject the manufacturer's claim?

Step 3: Compute the test value.

The standard deviation s must be squared in the formula.

Example 8-26: Nicotine Content

A cigarette manufacturer wishes to test the claim that the variance of the nicotine content of its cigarettes is 0.644. Nicotine content is measured in milligrams, and assume that it is normally distributed. A sample of 20 cigarettes has a standard deviation of 1.00 milligram. At α = 0.05, is there enough evidence to reject the manufacturer's claim?

Step 3: Compute the test value.

The standard deviation s must be squared in the formula.

$$\chi^2 = \frac{(n-1)s^2}{\sigma^2}$$

Example 8-26: Nicotine Content

A cigarette manufacturer wishes to test the claim that the variance of the nicotine content of its cigarettes is 0.644. Nicotine content is measured in milligrams, and assume that it is normally distributed. A sample of 20 cigarettes has a standard deviation of 1.00 milligram. At α = 0.05, is there enough evidence to reject the manufacturer's claim?

Step 3: Compute the test value.

The standard deviation s must be squared in the formula.

$$\chi^2 = \frac{(n-1)s^2}{\sigma^2} = \frac{(19)(1.00)^2}{0.644}$$

Example 8-26: Nicotine Content

A cigarette manufacturer wishes to test the claim that the variance of the nicotine content of its cigarettes is 0.644. Nicotine content is measured in milligrams, and assume that it is normally distributed. A sample of 20 cigarettes has a standard deviation of 1.00 milligram. At $\alpha = 0.05$, is there enough evidence to reject the manufacturer's claim?

Step 3: Compute the test value.

The standard deviation s must be squared in the formula.

$$\chi^2 = \frac{(n-1)s^2}{\sigma^2} = \frac{(19)(1.00)^2}{0.644} = 29.5$$

Example 8-26: Nicotine Content

Step 4: Make the decision.

Do not reject the null hypothesis, since the test value falls in the noncritical region.

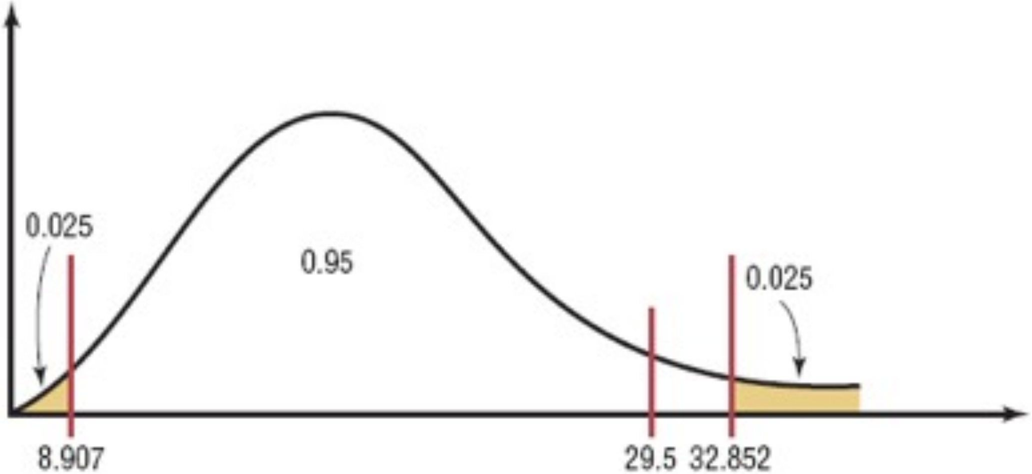
Step 5: Summarize the results.

There is not enough evidence to reject the manufacturer's claim that the variance of the nicotine content of the cigarettes is 0.644.

Example 8-26: Nicotine Content

Step 4: Make the decision.

Do not reject the null hypothesis, since the test value falls in the noncritical region.



Step 5: Summarize the results.

There is not enough evidence to reject the manufacturer's claim that the variance of the nicotine content of the cigarettes is 0.644.

Example 8-26: Nicotine Content

Step 4: Make the decision.

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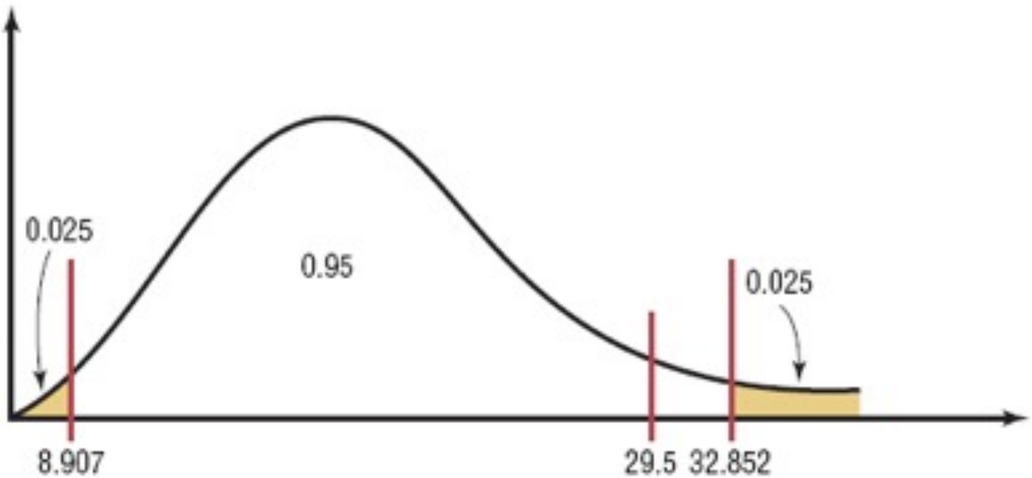

Example 8-26: Nicotine Content

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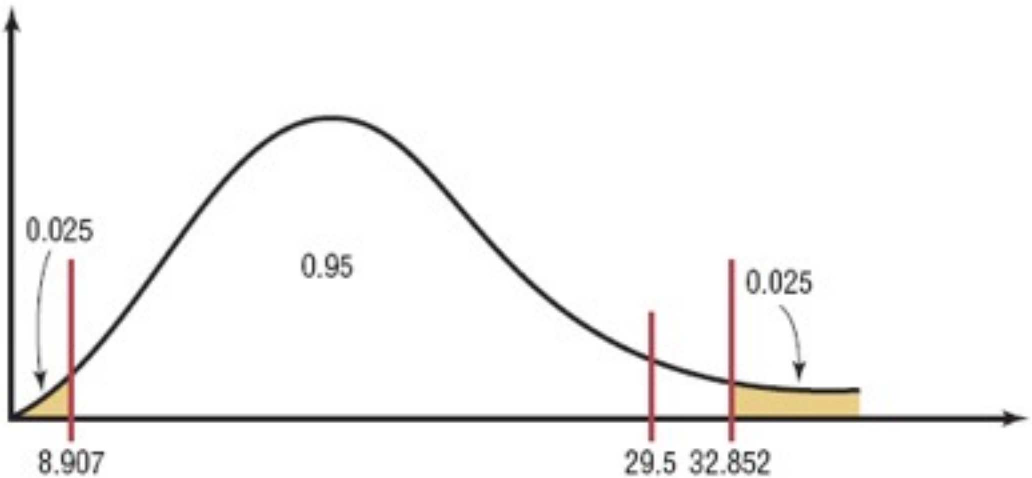

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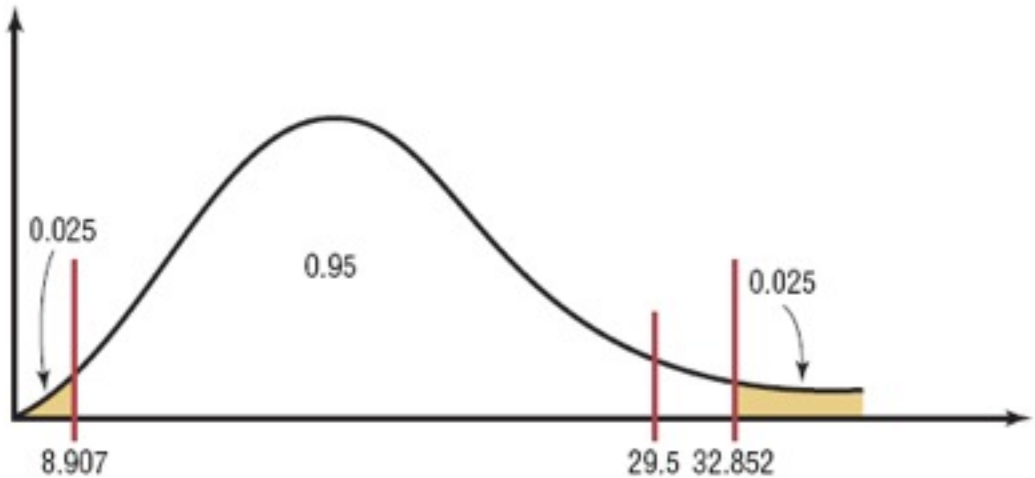

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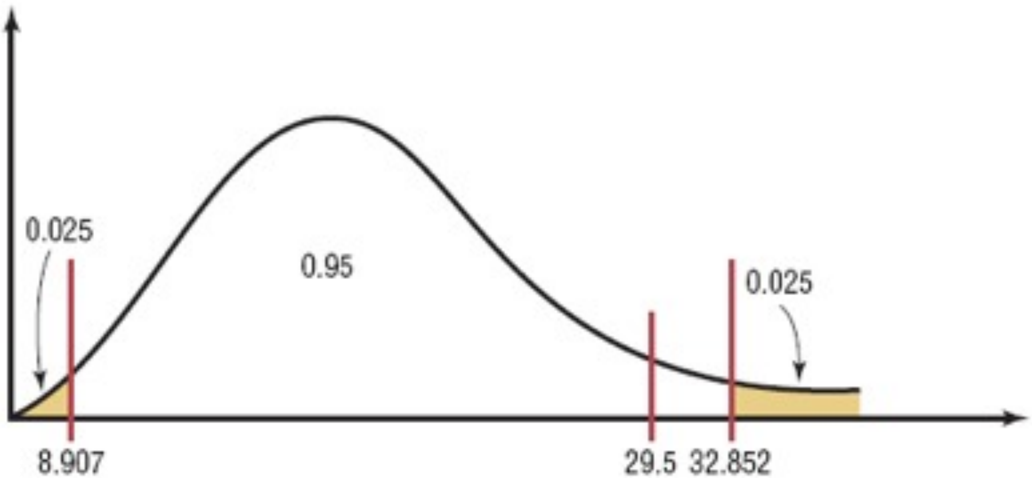

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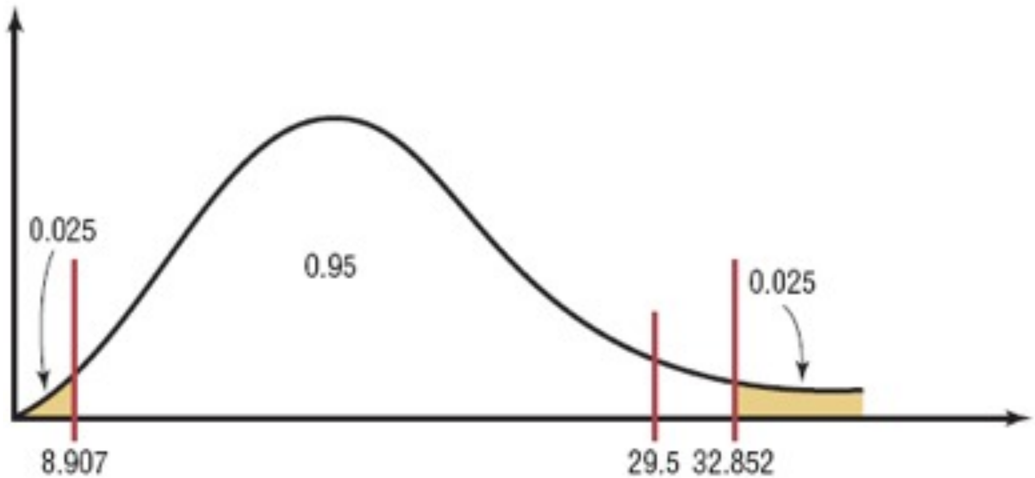

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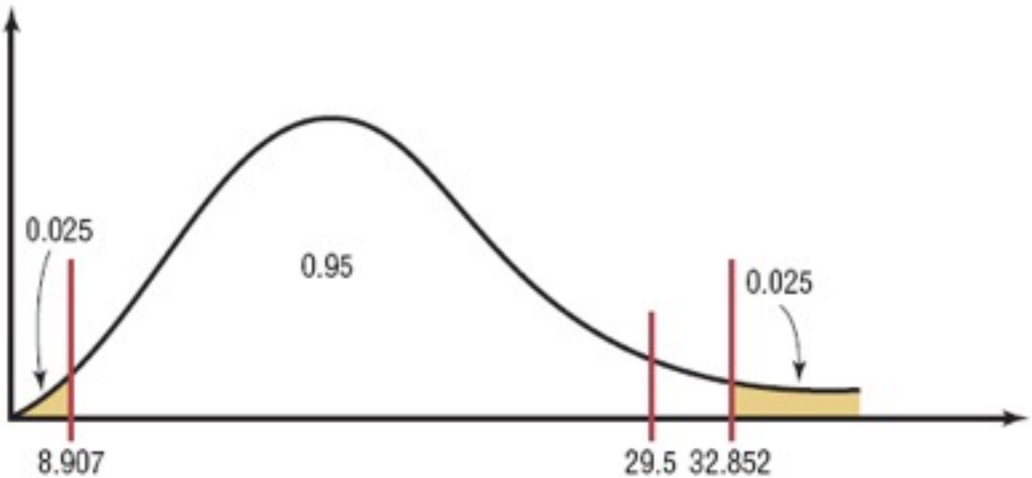

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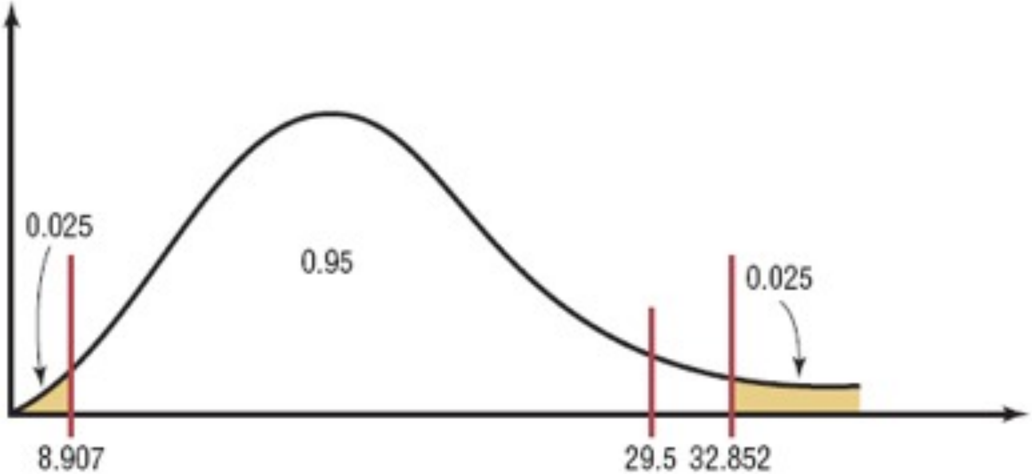

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Example 8-26: Nicotine Content

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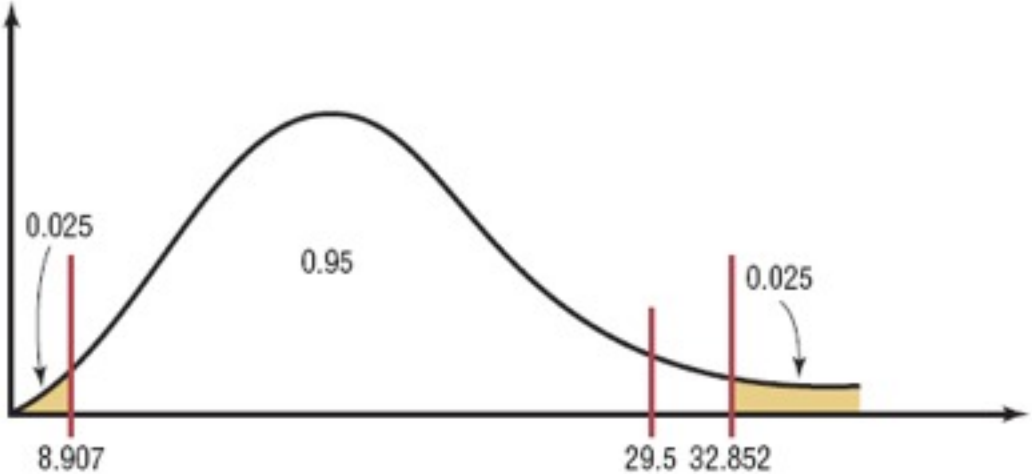

Step 5: Summarize the results.

There is not enough evidence to reject the manufacturer's claim that the variance of the nicotine content of the cigarettes is 0.644.