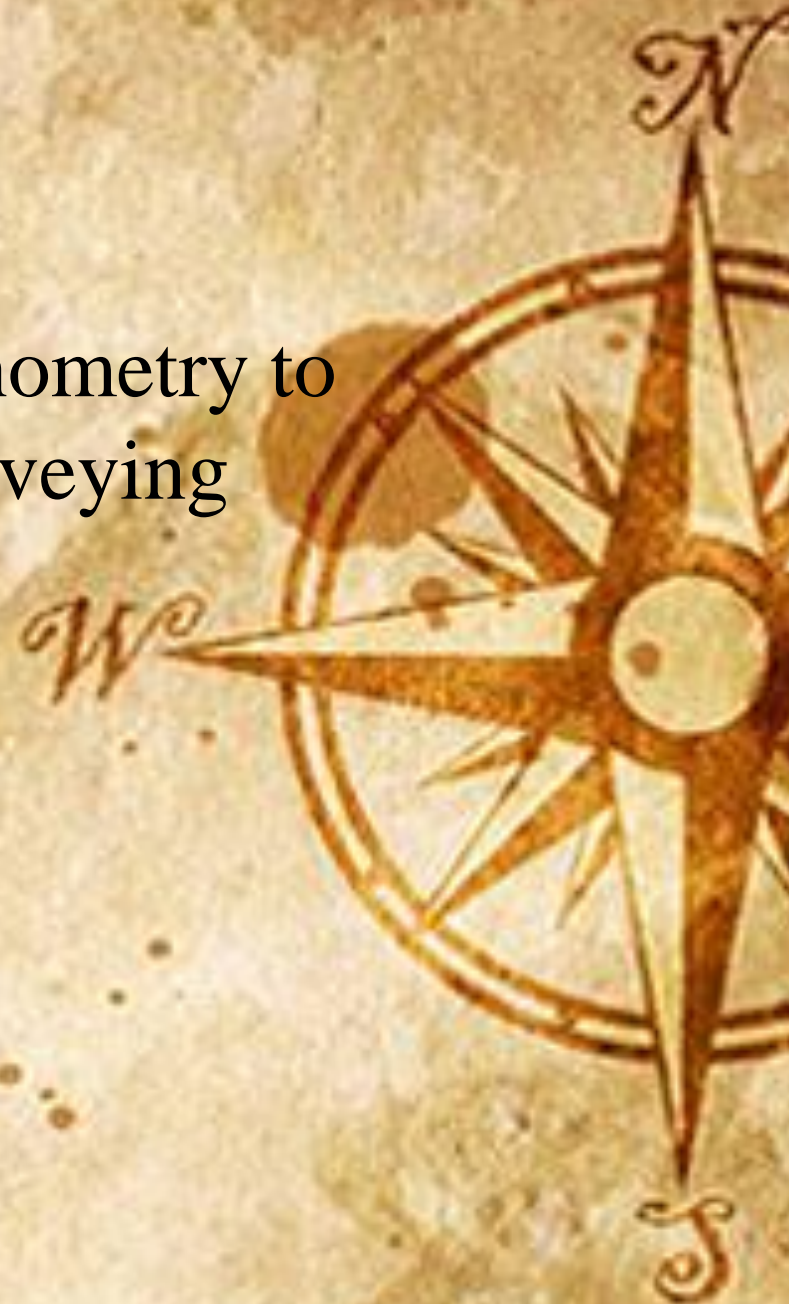
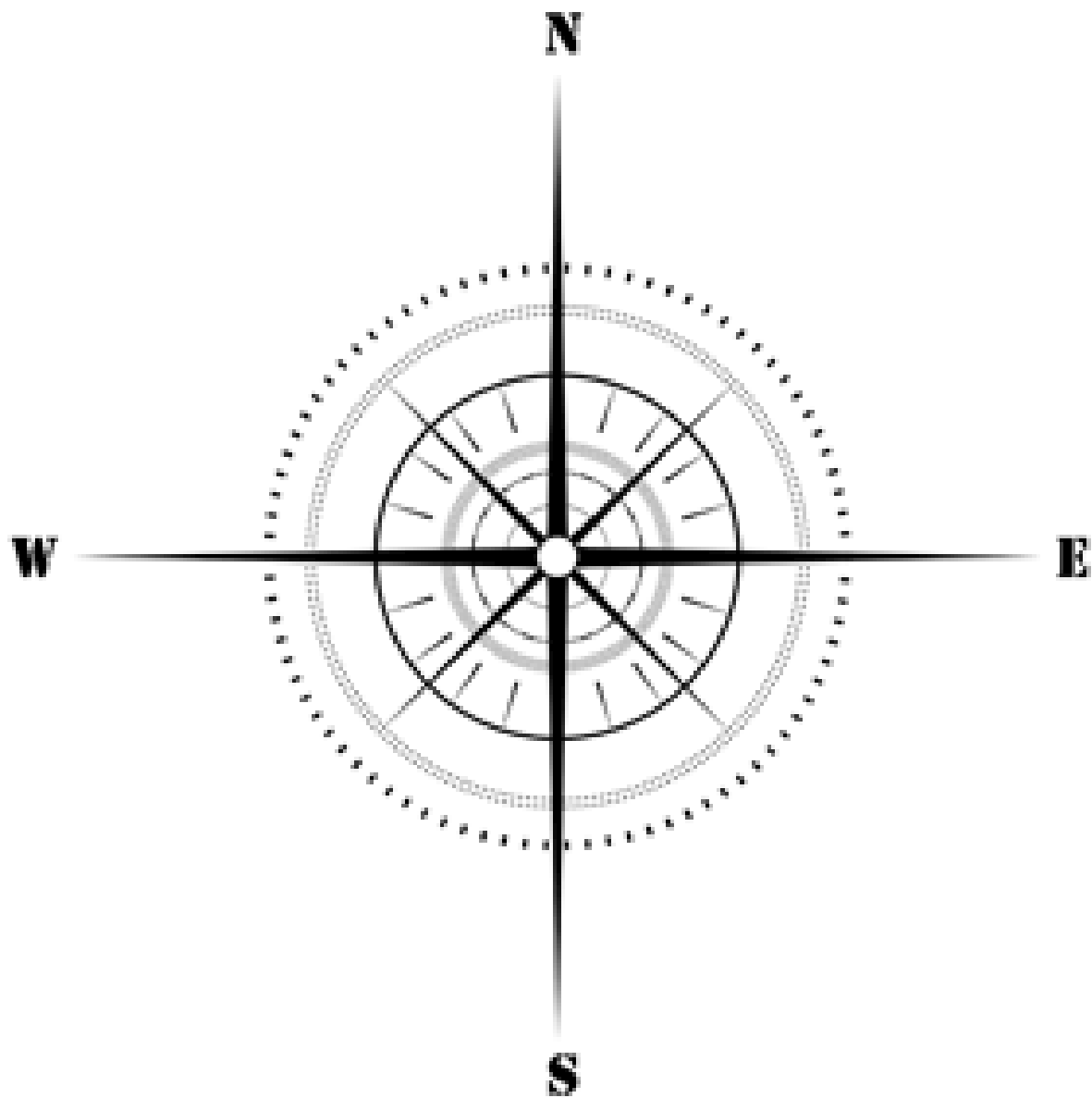


# Sec 9.5

## Applications of Trigonometry to Navigation and Surveying







# Which direction?

- In basic Trig... standard position:

Figure 1

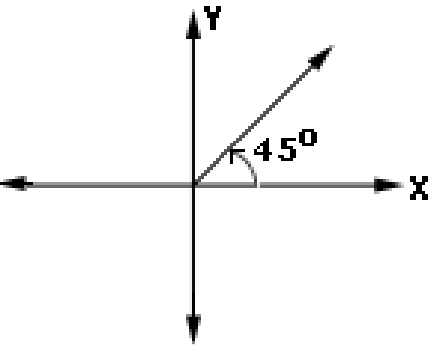


Figure 2

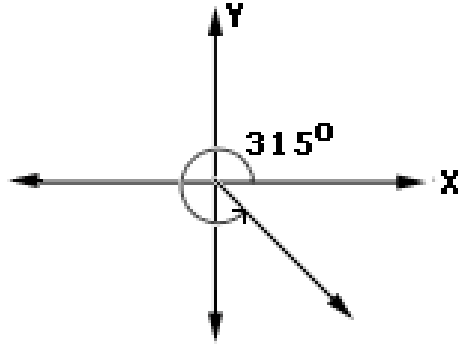


Figure 3

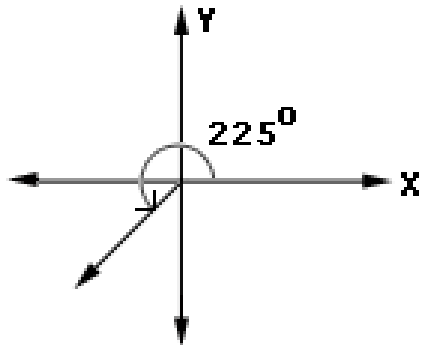
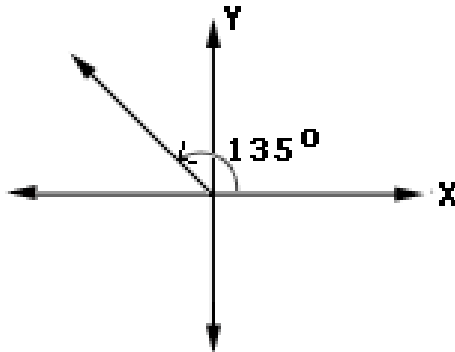


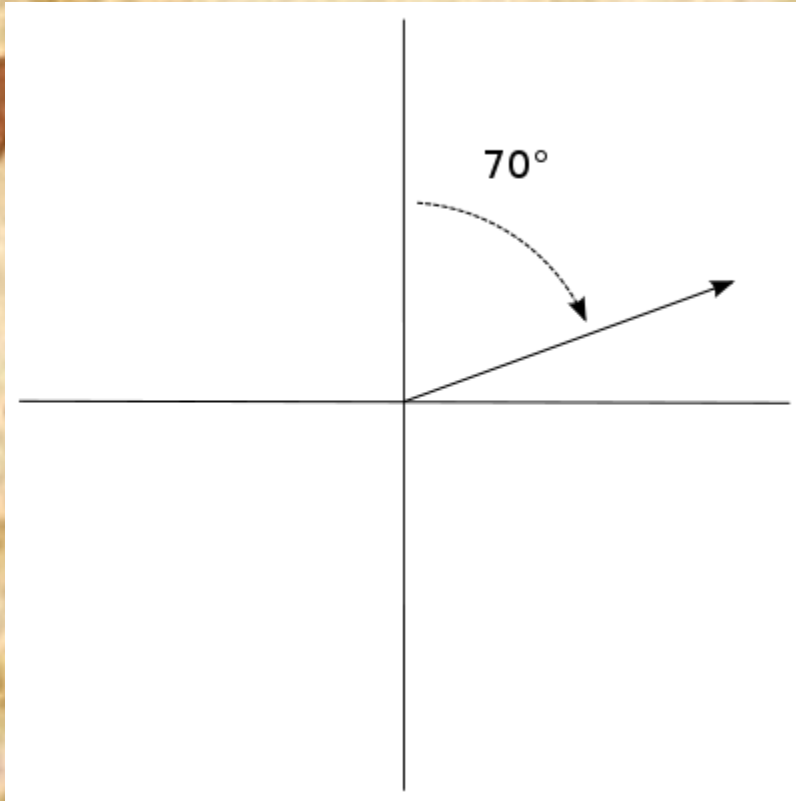
Figure 4



Start on the x-axis.  
Counter clockwise

# Which direction?

- Navigation... used by ships, planes etc.



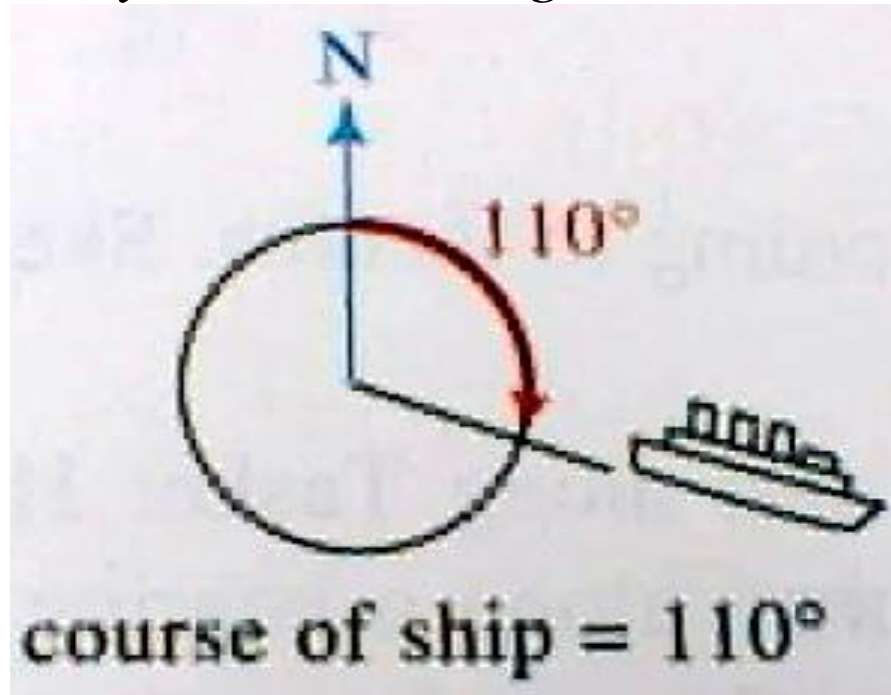
Start on the y-axis.  
Clockwise  
Given using 3 digits



# 9.5 Applications of Trigonometry to Navigation and Surveying

## *Objective*

*To use trigonometry to solve navigation and surveying problems.*

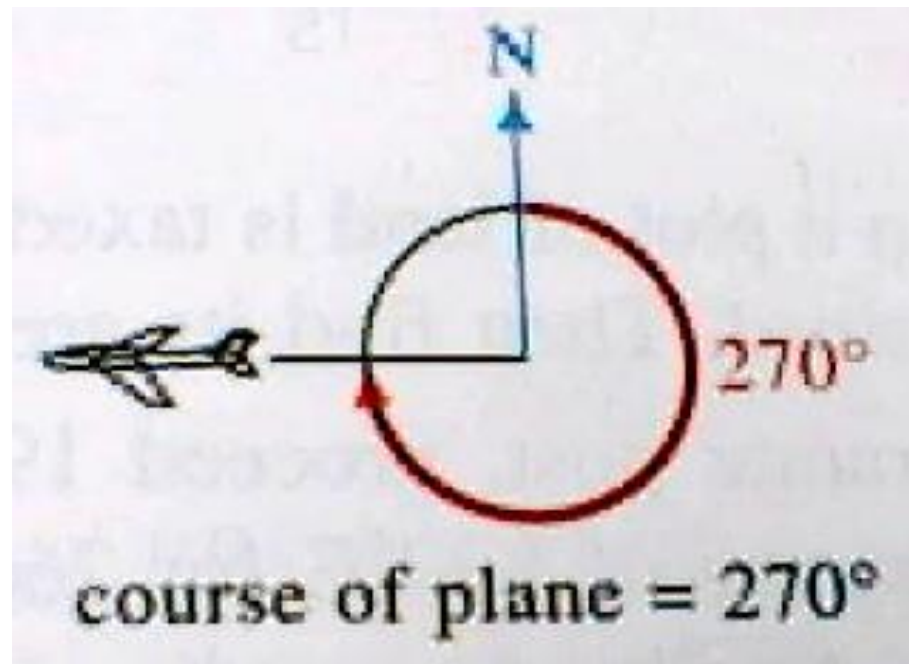


The course of a ship or plane is the  $\angle$ , measured clockwise, from the north direction to the direction of the ship or plane.

# 9.5 Applications of Trigonometry to Navigation and Surveying

## *Objective*

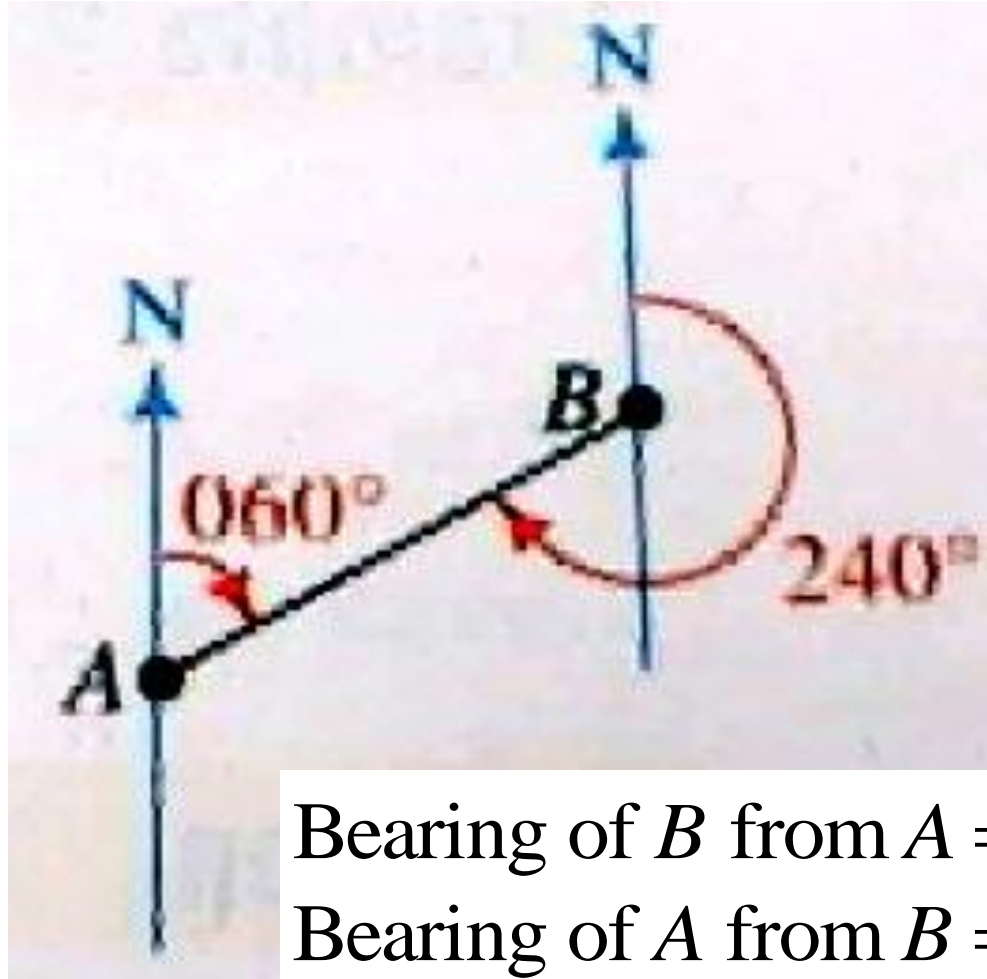
*To use trigonometry to solve navigation and surveying problems.*



The course of a ship or plane is the  $\angle$ , measured clockwise, from the north direction to the direction of the ship or plane.

# Objective

*To use trigonometry to solve navigation and surveying problems.*



The course of a ship or plane is the  $\angle$ , measured clockwise, from the north direction to the direction of the ship or plane.



Example 1. A ship proceeds on a course of  $300^\circ$  for 2 hours at a speed of 15 knots (1 knot = 1 nautical mile per hour). Then it changes course to  $230^\circ$ , continuing at 15 knots for 3 more hours. At that time, how far is the ship from its starting point?

**Make a diagram**



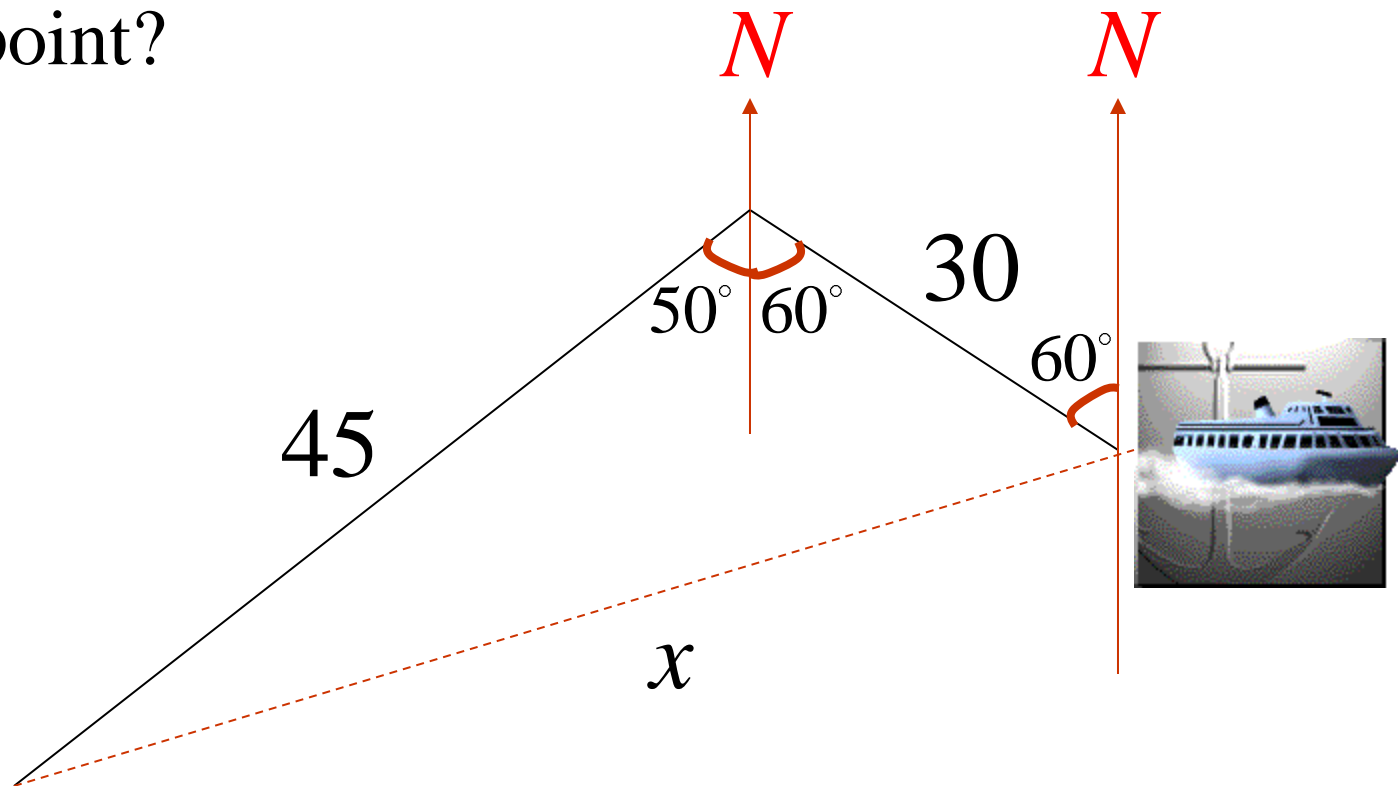


Always measure clockwise

A ship proceeds on a course of  $300^\circ$  for 2 hours at a speed of 15 knots (1 knot = 1 nautical mile per hour).

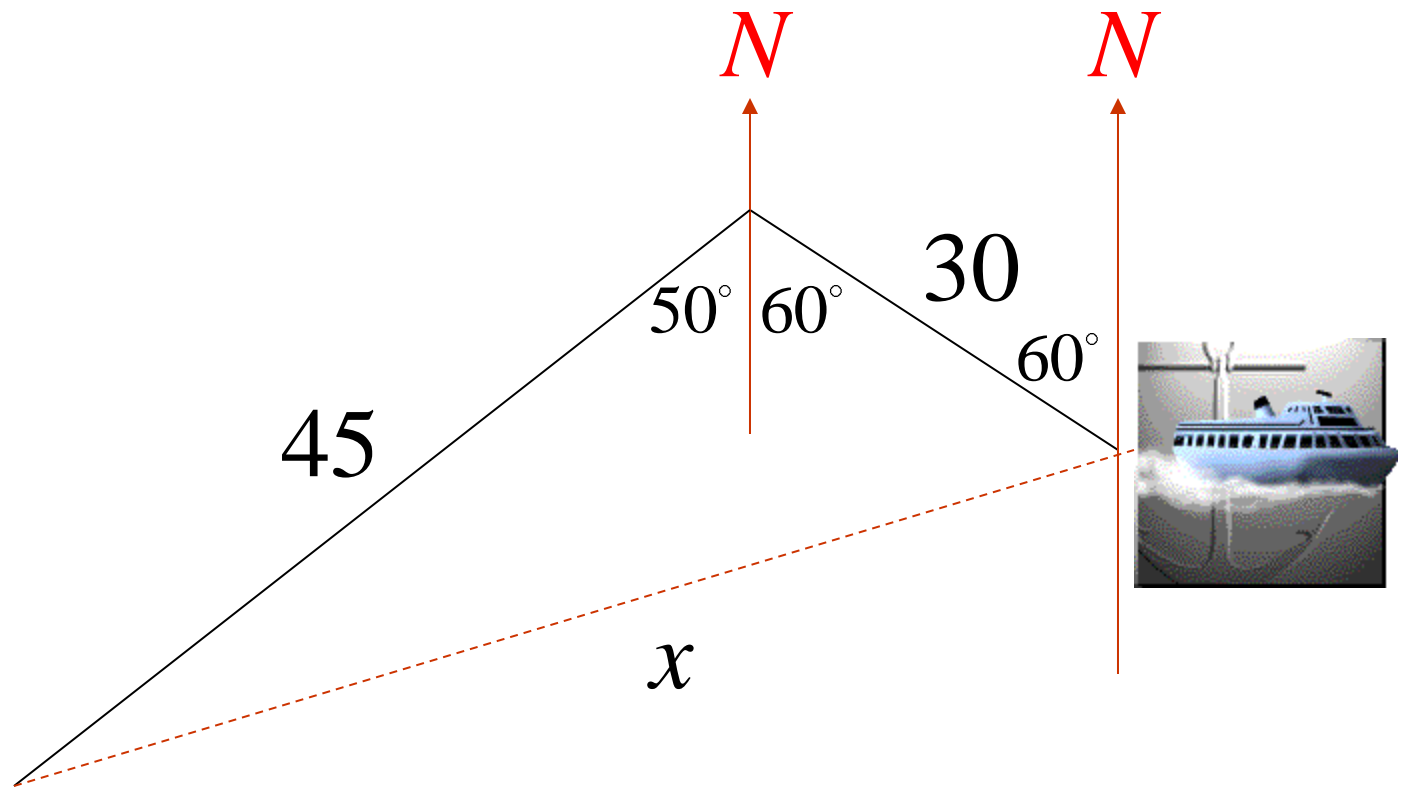
Then it changes course to  $230^\circ$ , continuing at 15 knots for 3 more hours. At that time, how far is the ship from the starting point?

Law of  
Cosines



$$(OPP)^2 = (ADJ_1)^2 + (ADJ_2)^2 - 2(ADJ_1)(ADJ_2)\cos(\angle)$$

# Law of Cosines

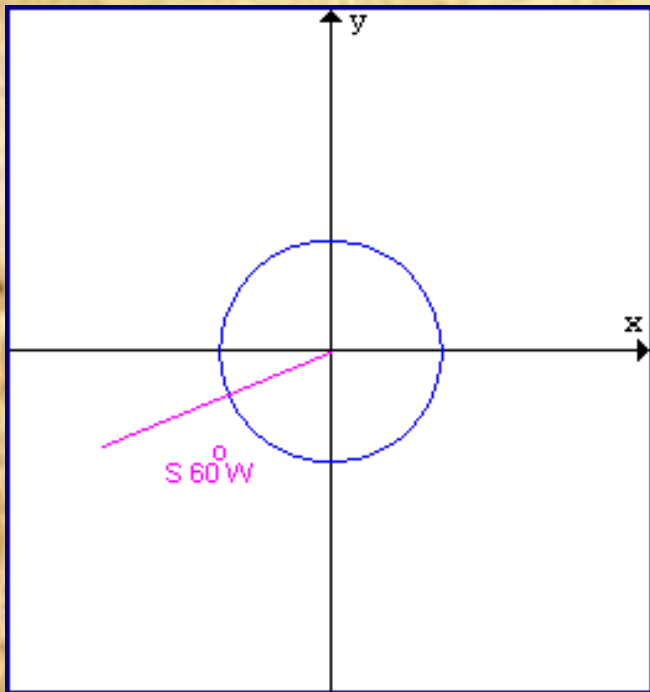


$$(OPP)^2 = (ADJ_1)^2 + (ADJ_2)^2 - 2(ADJ_1)(ADJ_2)\cos(\angle)$$



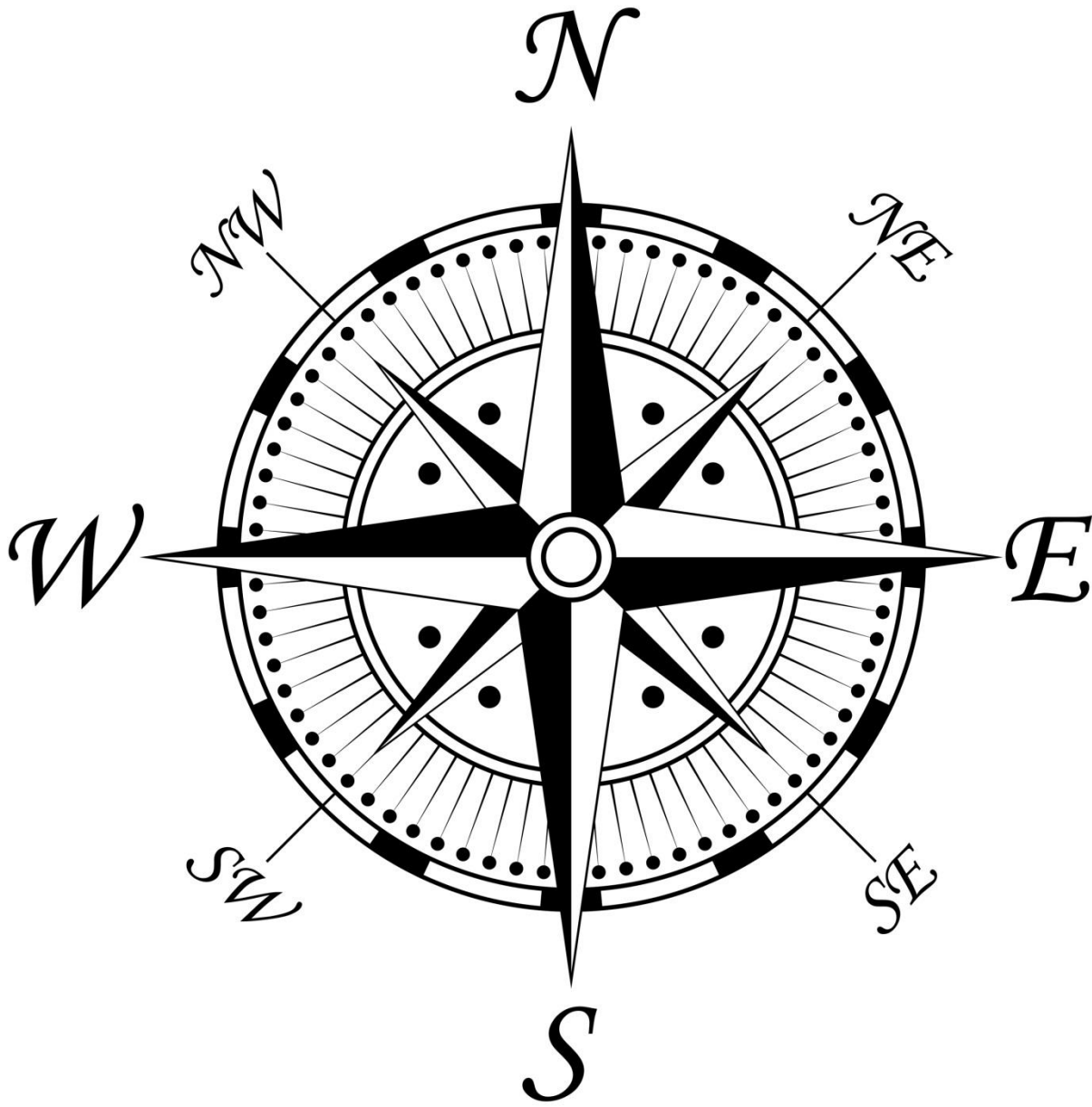
# Which direction?

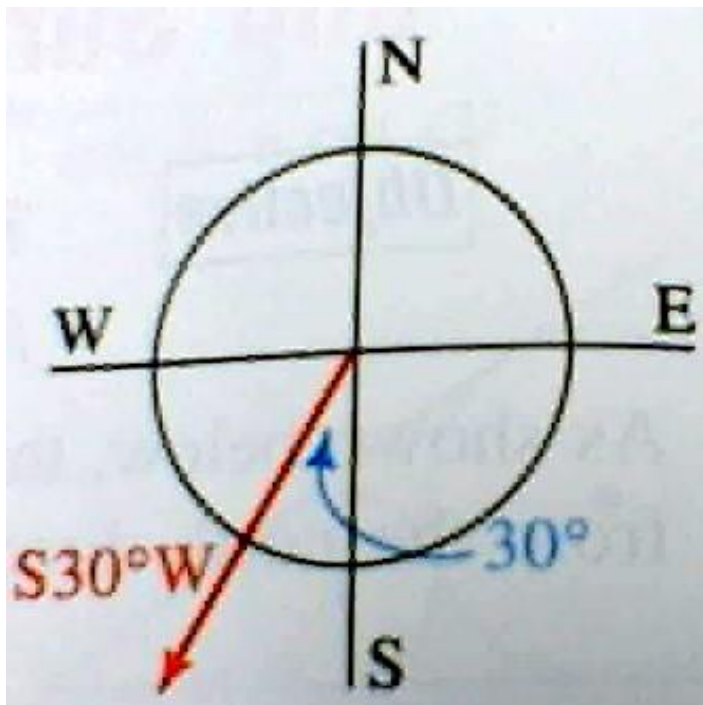
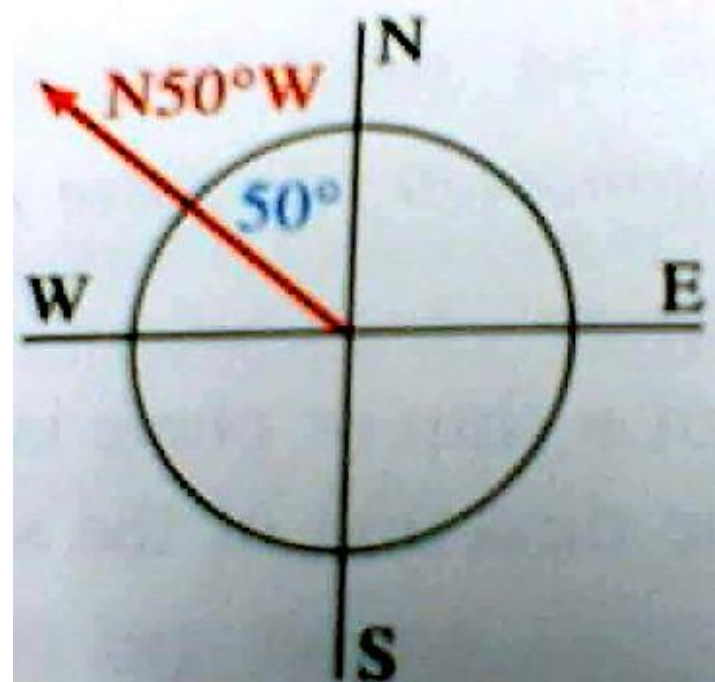
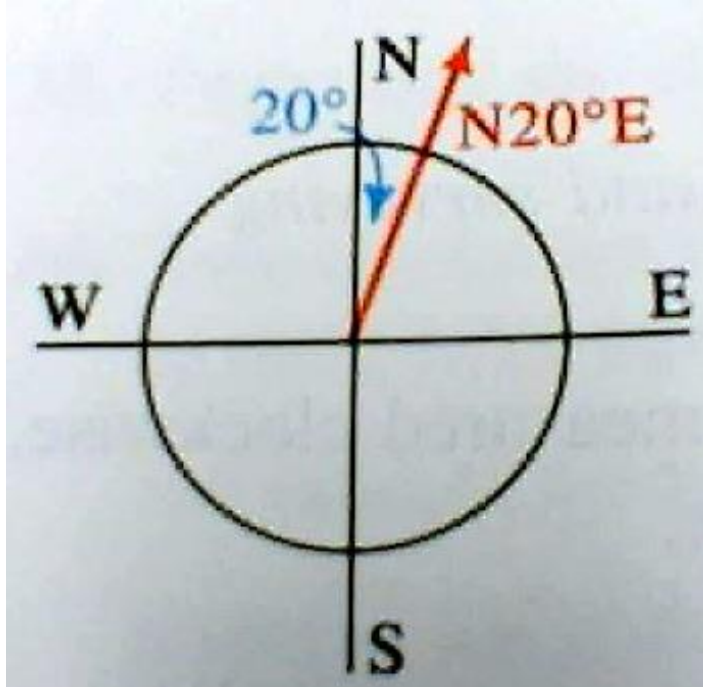
- In surveying, a compass reading is usually given as an acute angle from the north-south line toward the east or west line.



- a) Start on the y-axis.
- b) Clockwise
- c) The angle is always acute.

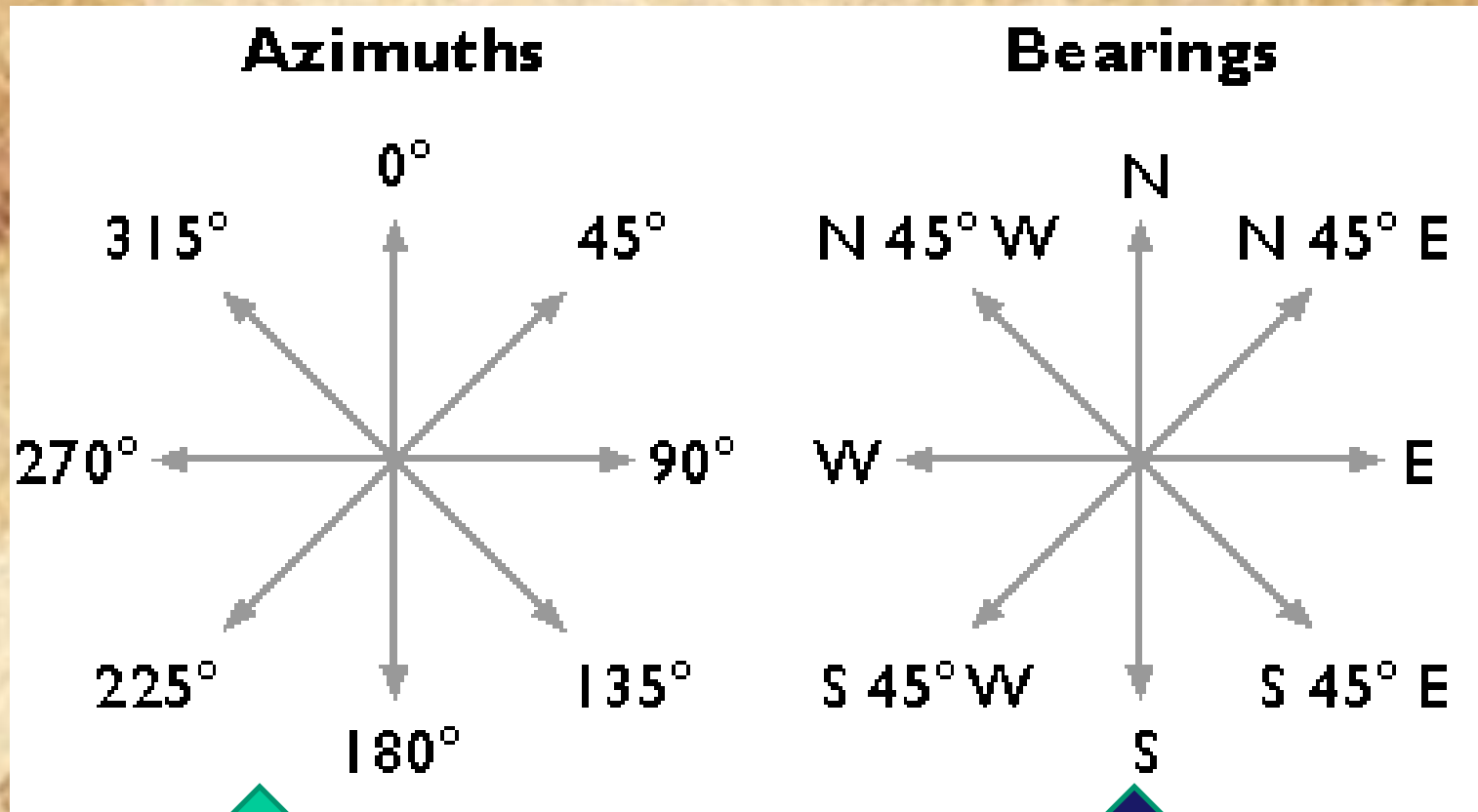






In surveying, a compass reading is usually given as an acute  $\angle$  from the north-south line toward the east or west.

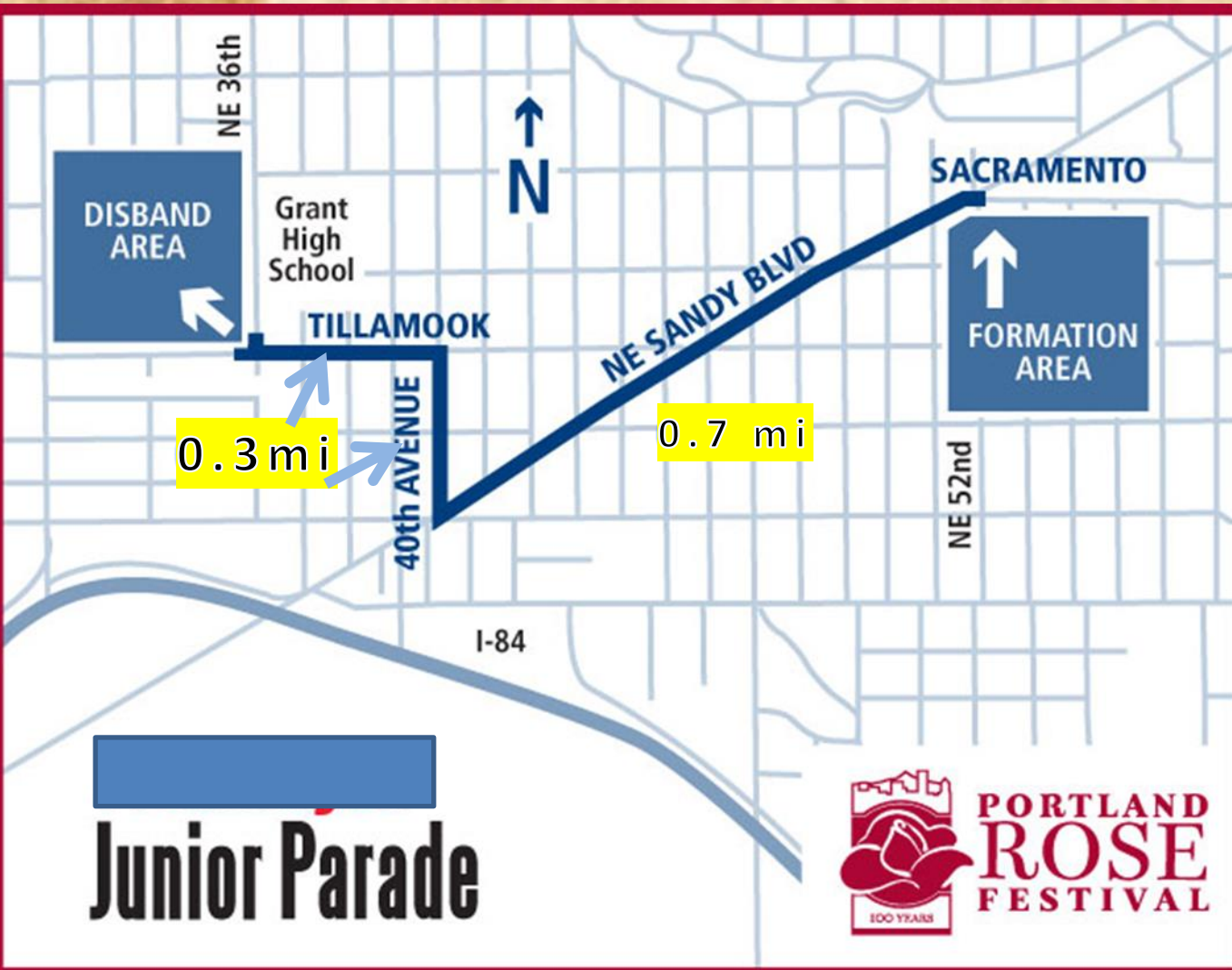




  
**Navigation**

  
**Surveying**

NE Sandy and NE 40<sup>th</sup> meet at approx 58 degree angle.



- Give directions from the Formation Area to the disband Area.
- Using navigation system.
  - Using the survey method.



# Basic Hints and rules

- Make a diagram... give yourself drawing space *all* around the diagram.
- Although drawing to scale might be hard, come as close to a scale as possible.
- Write all the given information on your diagram.



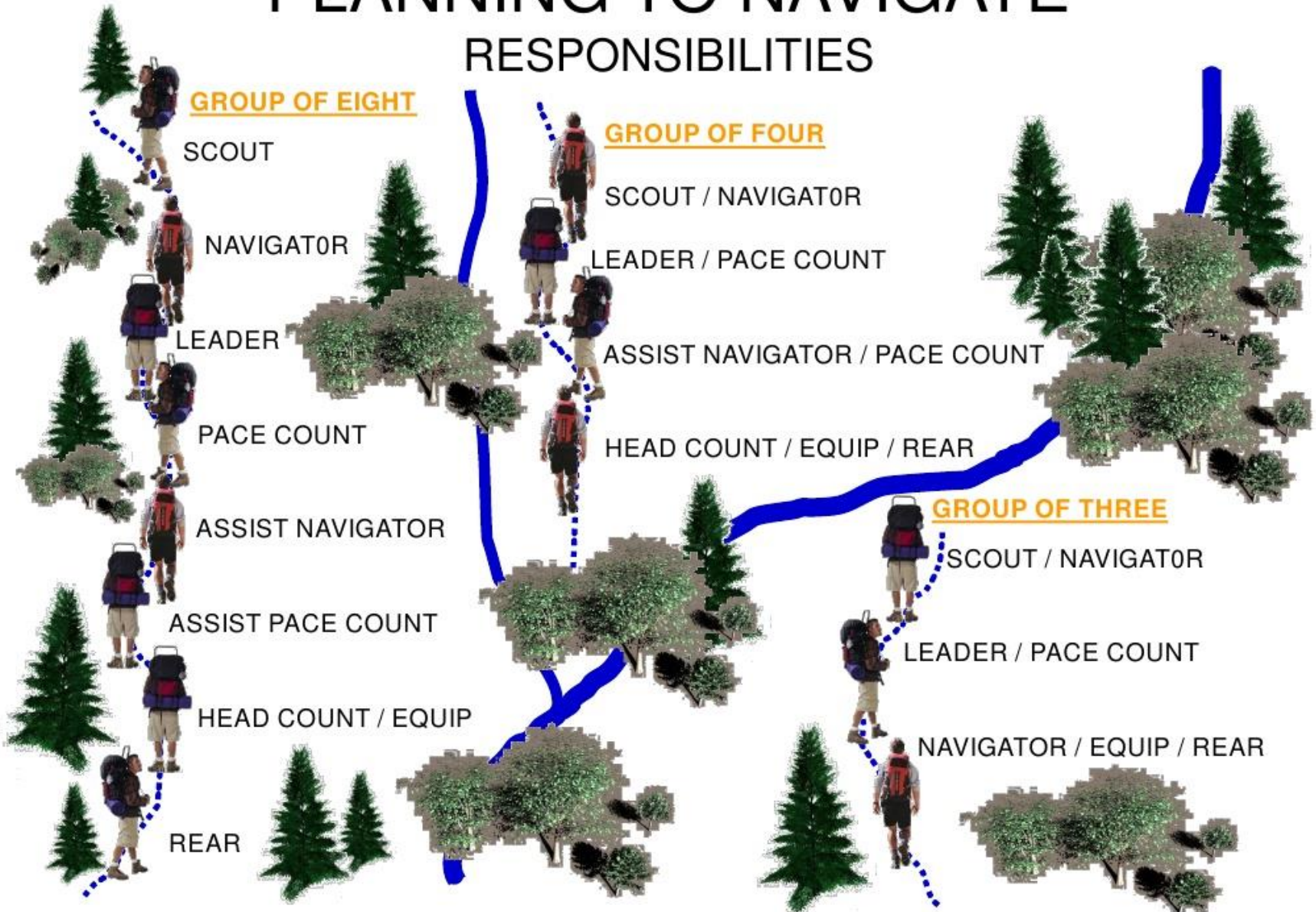
# Basic Hints and rules

- Include a lightly drawn x and y axes at each point.
- Find as many angles and sides as you can.
- Apply as many geometry rules as you can.



# PLANNING TO NAVIGATE

## RESPONSIBILITIES

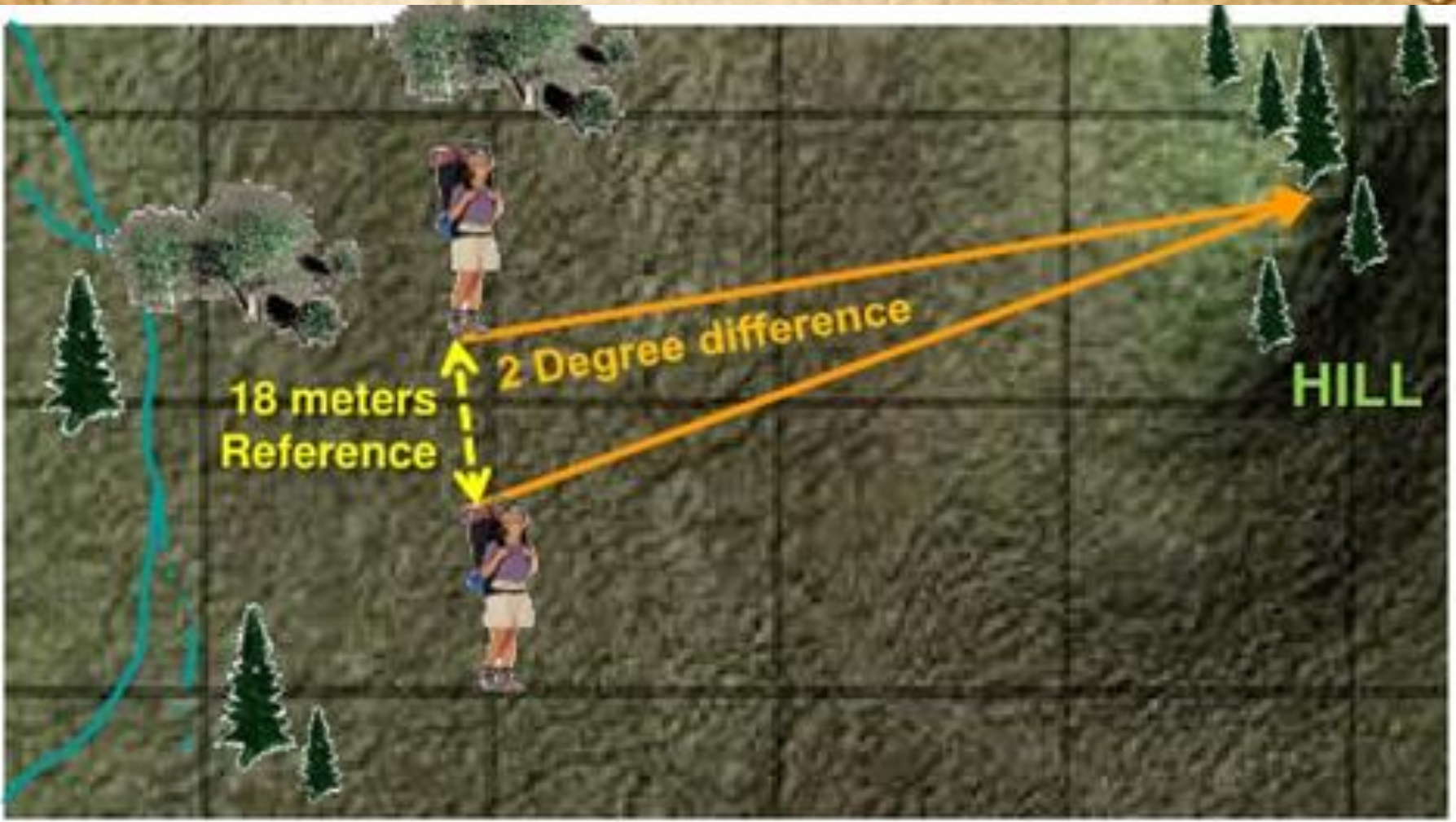




18 meters  
Reference

2 Degree difference

HILL





# Camping:

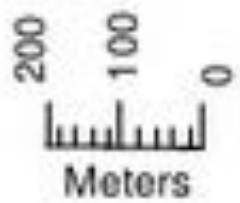
Give direction from the camp site to each of 4 points of interest:

- a) The river
- b) The lake
- c) To the tower
- d) The hill

*Generate clarifying questions!*



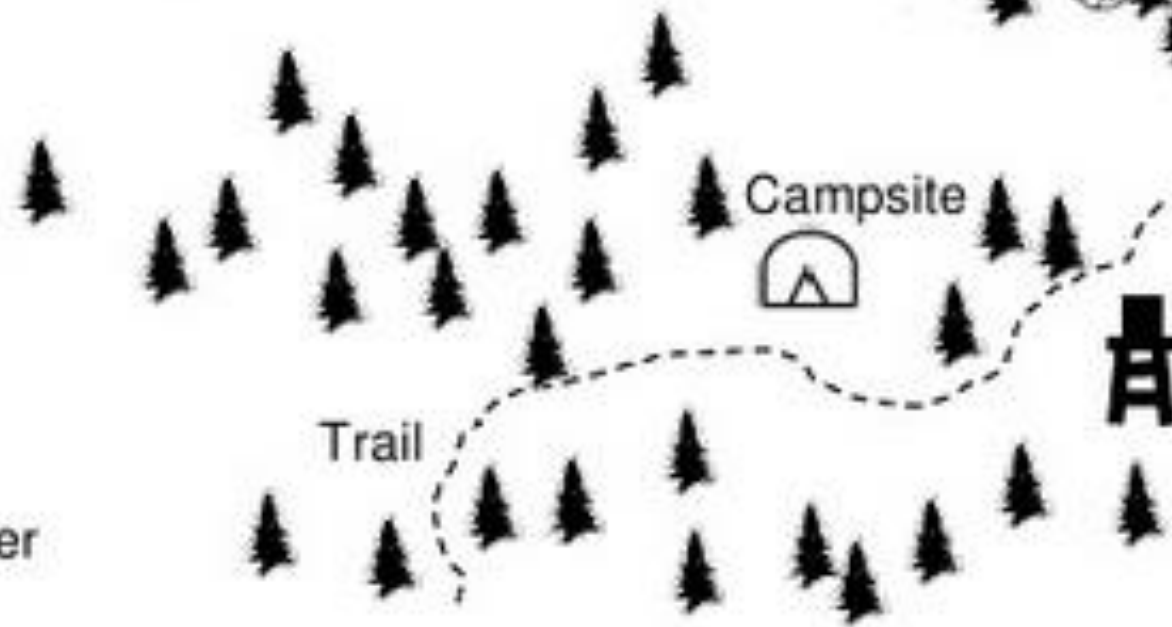
Hill



Magnetic North



Lake



Campsite

Trail



Tower



River



Example 2. Very often a plot of land is taxed according to its area. Sketch the plot of land described. Then find its area.

From a granite post, proceed 195 ft east along Tasker Hill Road, then along a bearing of  $S32^\circ E$  for 260 ft, then along a bearing of  $S68^\circ W$  for 385 ft, and finally along a line back to the granite post.

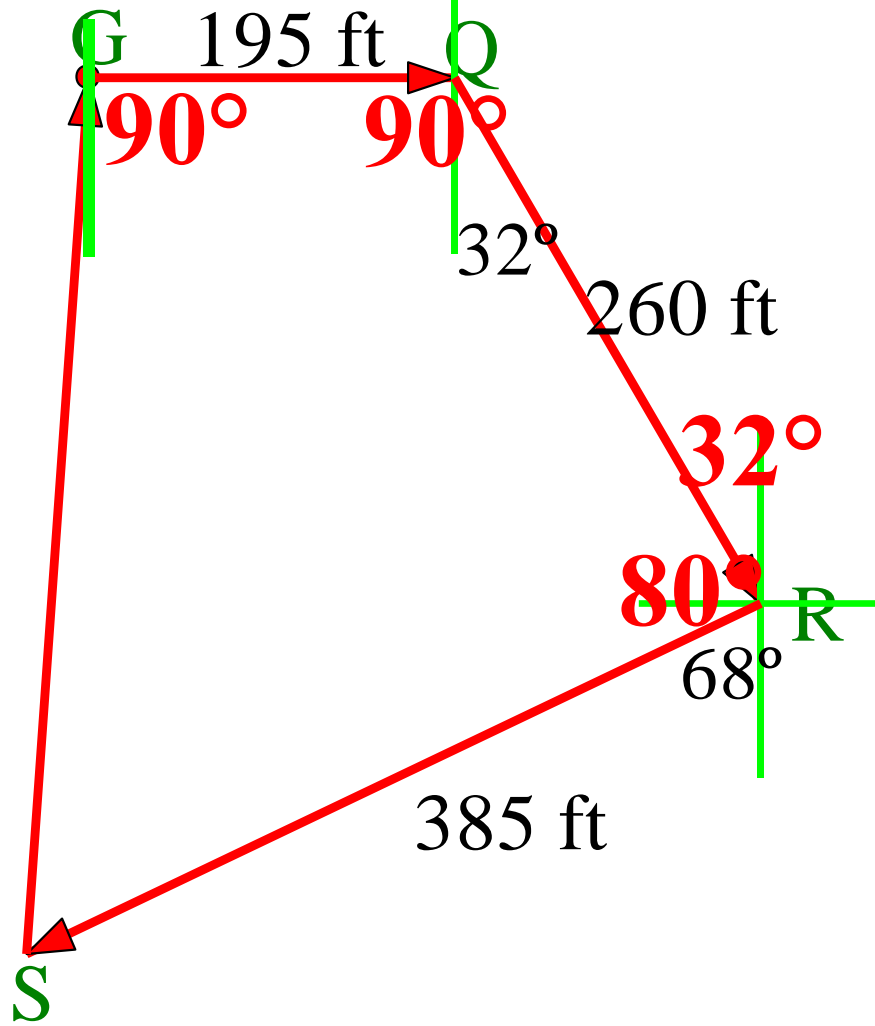
With these types of problems, a careful diagram is essential.

Next slide will demonstrate all the steps.

Make sure to have your geometric tools and math wits about you!



# Takser Hill Road

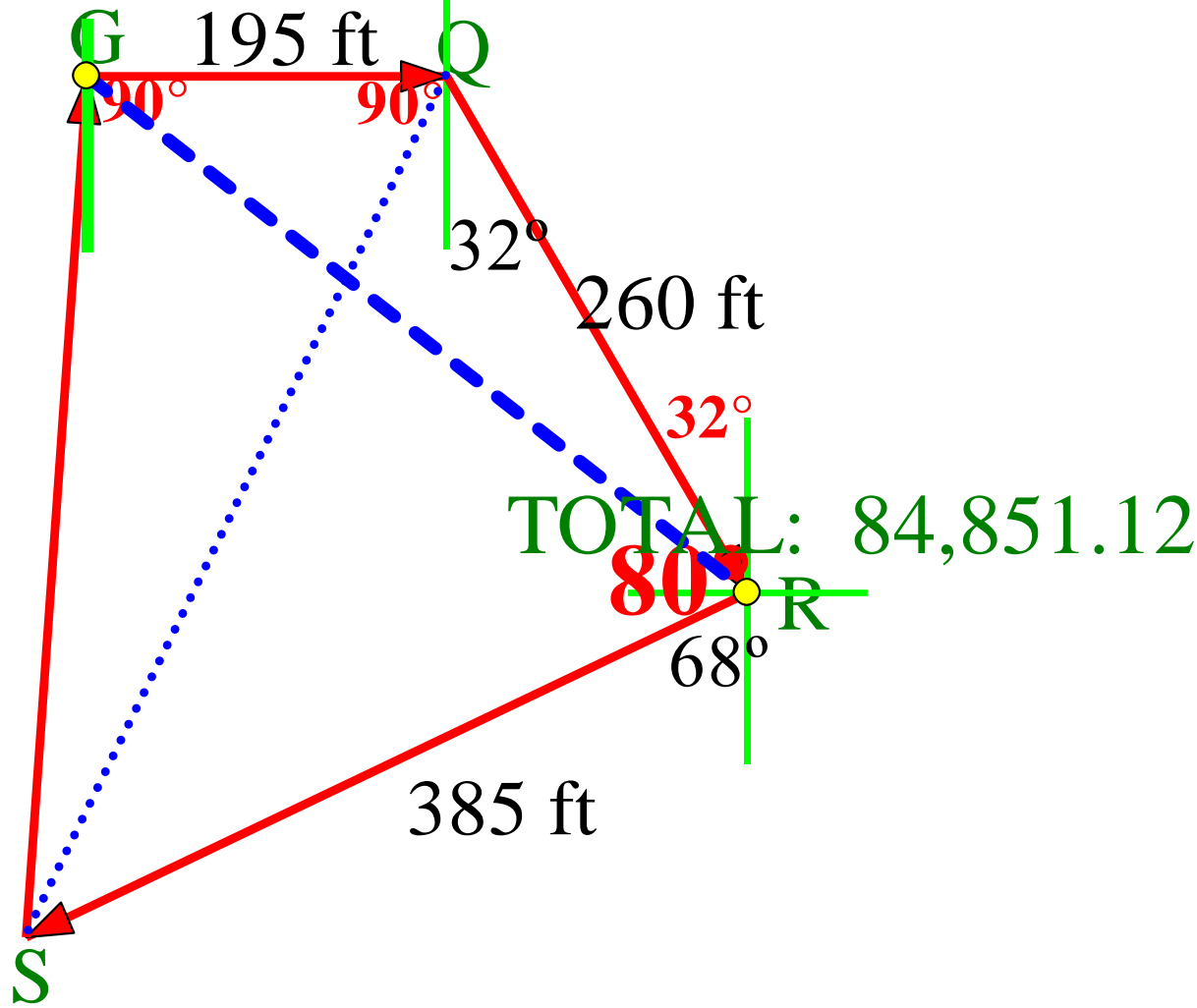




area of  $\triangle GQS=35,4$

Takser Hill Road

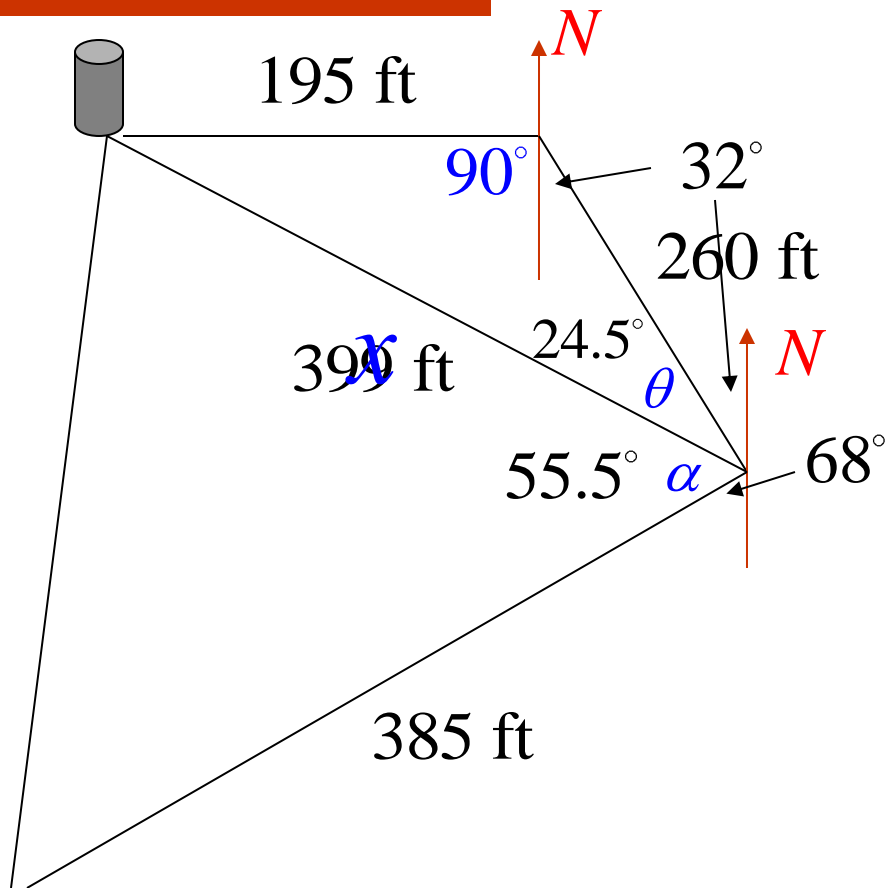
area of  $\triangle GRS=49,2$



A plot of land is taxed according to its area. Sketch the plot of land described, then find its area.

$$k = \frac{1}{2} ab \sin C$$

From a granite post, proceed 195 ft east along Tasker Hill Road, then along a bearing of S32°E for 260 ft, then along a bearing of S68°W for 385 ft, and finally along a line back to the granite post.

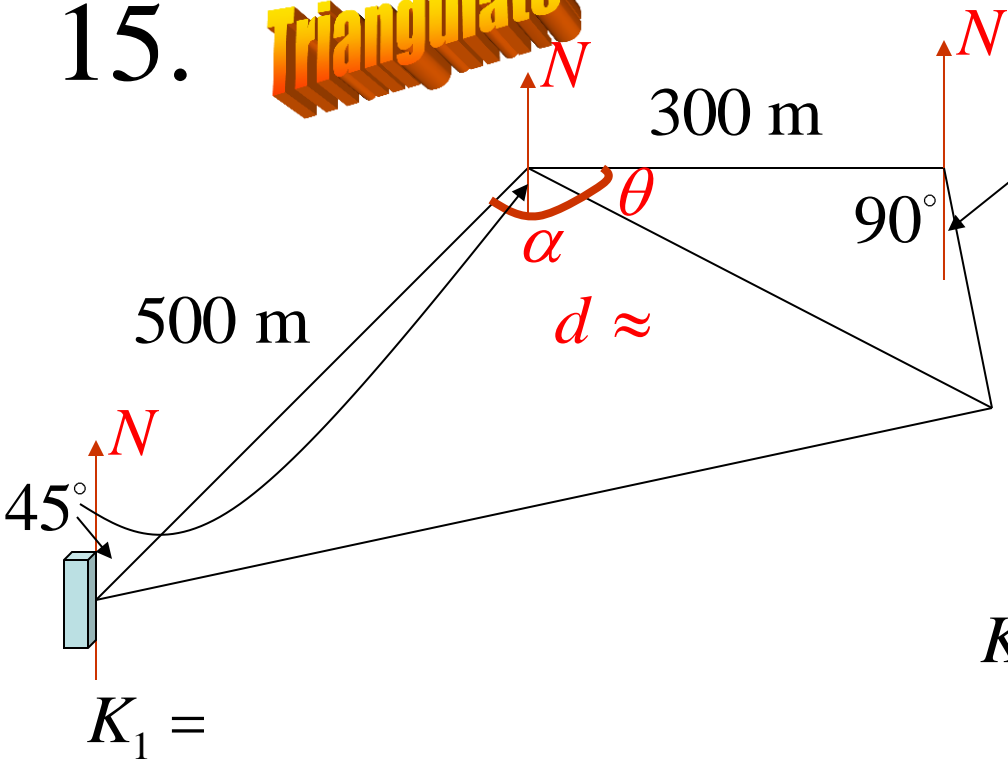


500 m NE;

15.

Triangulate

Law of Sines



$$\frac{\sin \theta}{200} = \frac{\sin \alpha}{500}$$

$$m\angle \alpha =$$

$$K_2 =$$

$$K_1 =$$

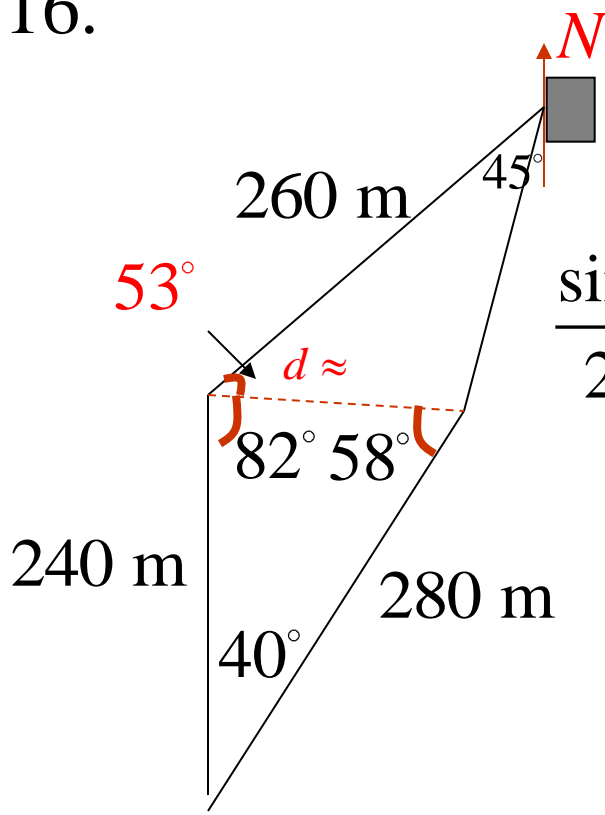
$$d^2 =$$

Area ≈



260 m SW;

16.



$K =$

$$\frac{\sin \theta}{240} = \frac{\quad}{\quad}$$

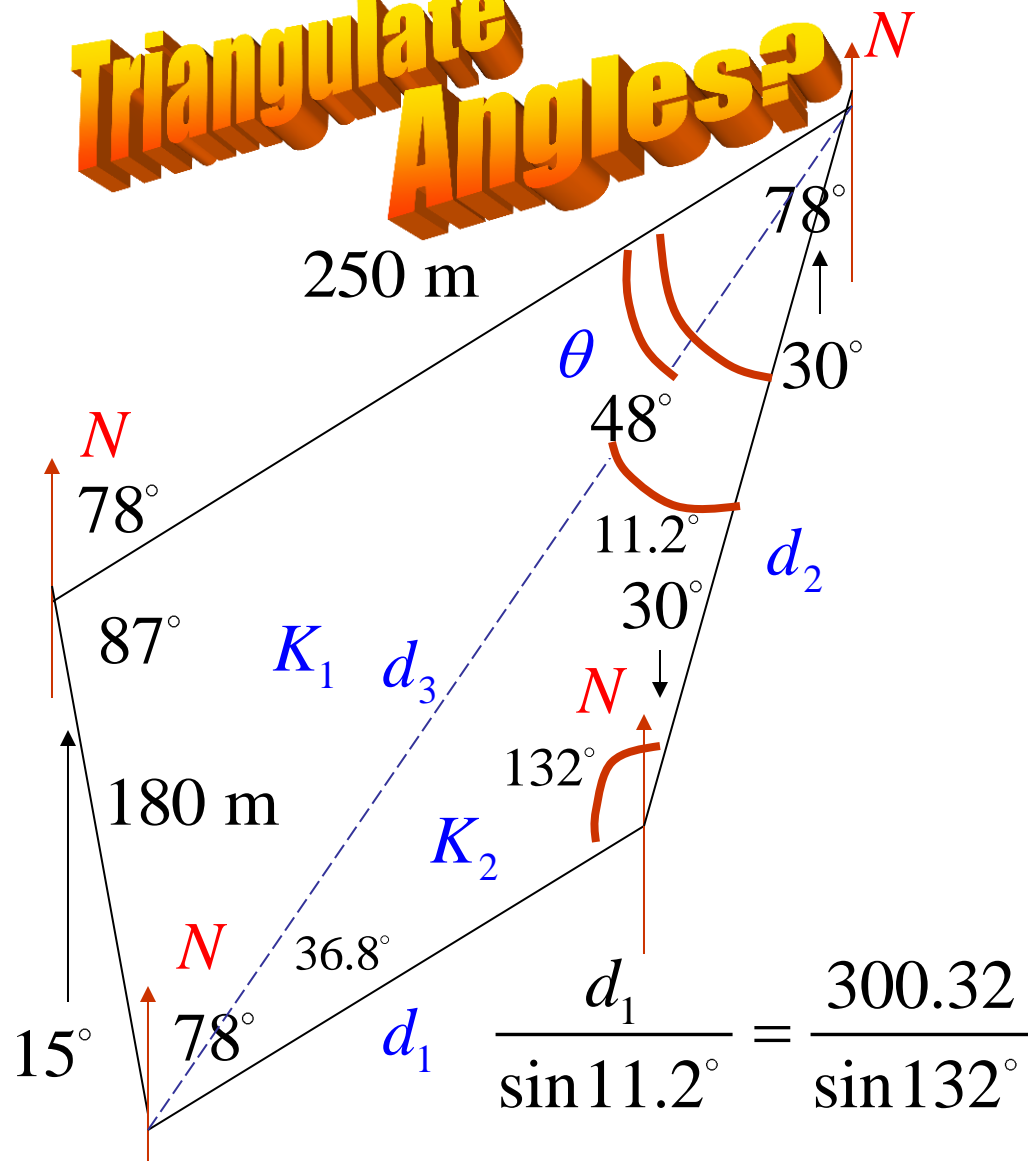
$d^2 =$

$K =$

*Area  $\approx$*

# Presentation

## Triangulate Angles?



## Law of Sines

$$\frac{\sin \theta}{180} = \frac{\sin 87^\circ}{300.32}$$

$$(d_3)^2 =$$

$$\frac{d_1}{\sin 11.2^\circ} = \frac{300.32}{\sin 132^\circ}$$

$$K_1 + K_2 \approx$$

- [click here for a sample test](#)



# Homework:

- **Sec 9.5**  
**written**  
**exercises**
- **7-13 odds;**  
**15, 16, 17**

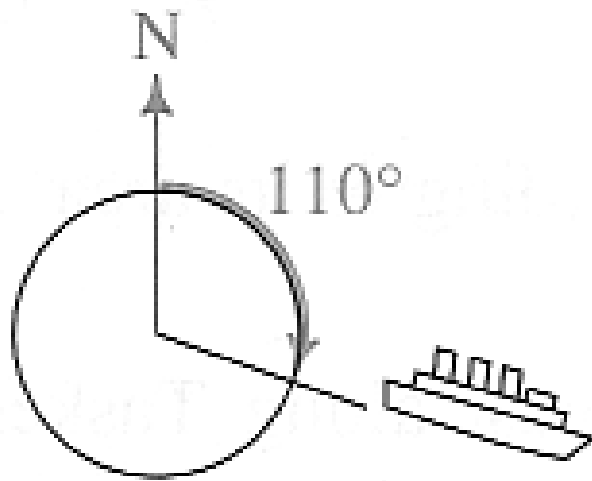
## **In class Exit Slip:**

- **Page 353 Problem #17.**
- **Only full solutions will be considered.**

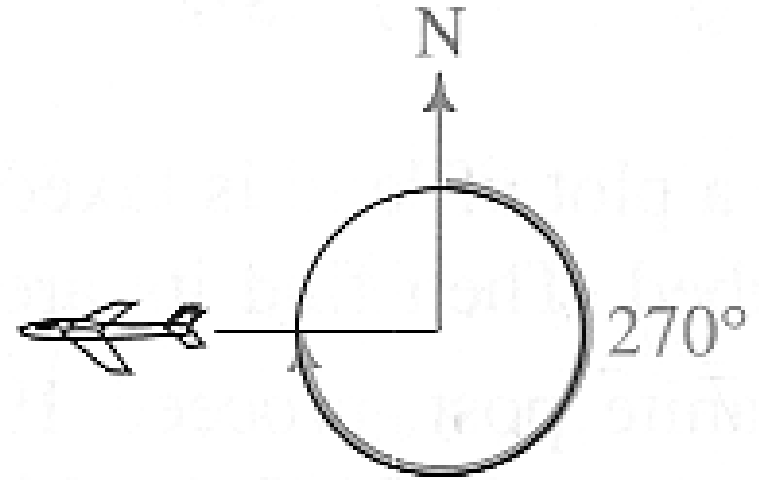
If you were absent, see Navi for make up Exit Slip.

# Applications of Trig to Navigation and Surveying

The course of a ship or plane is the angle, measured clockwise, from the north direction to the direction of the ship or plane.

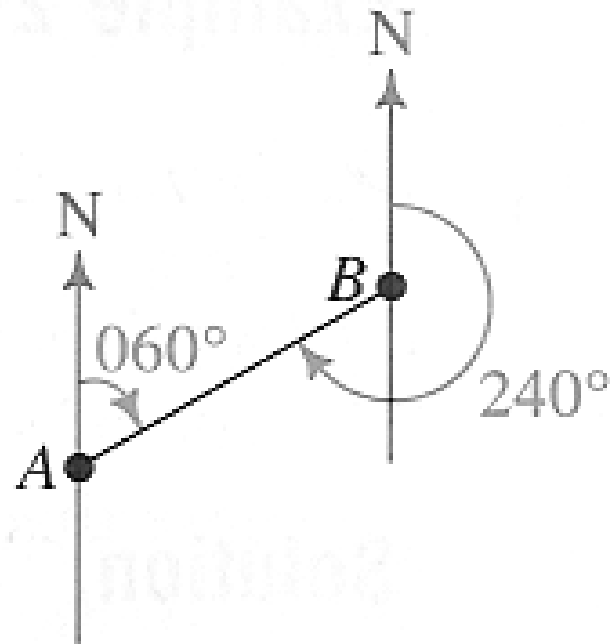


course of ship =  $110^\circ$



course of plane =  $270^\circ$

As shown, the compass bearing of one location from another is measured in the same way. Note that compass bearings and courses are given with three digits, such as  $060^\circ$  rather than  $60^\circ$



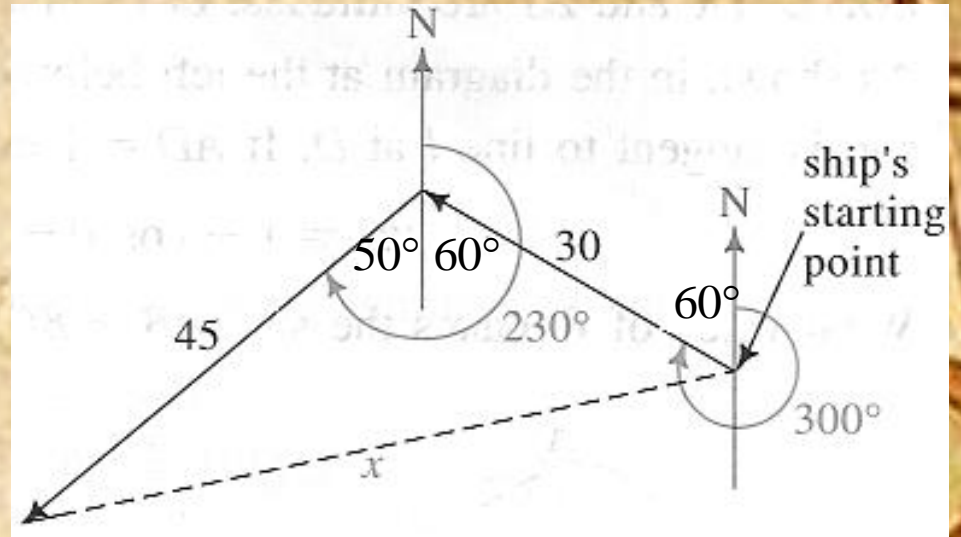
bearing of  $B$  from  $A = 060^\circ$   
bearing of  $A$  from  $B = 240^\circ$





G.

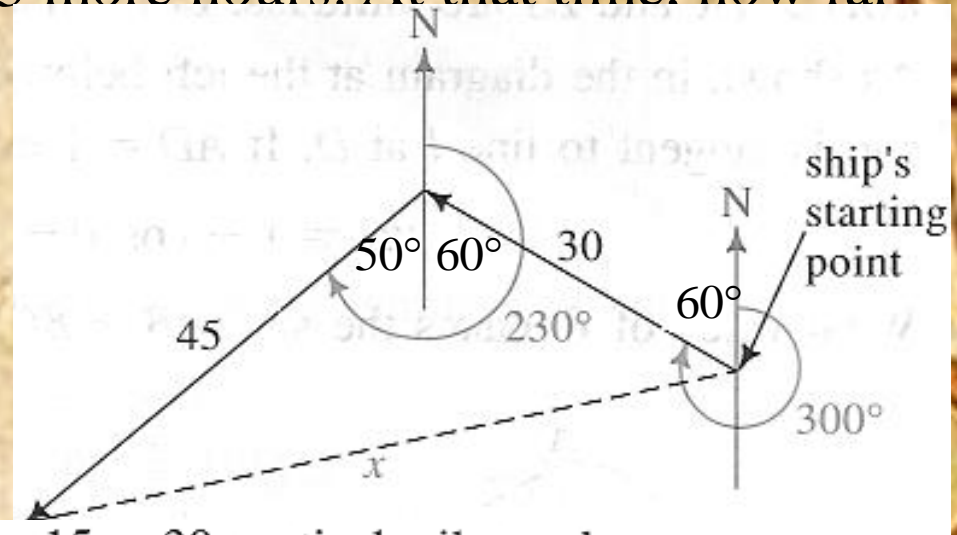




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Make a diagram



The ship travels first along a path of length  $2 \cdot 15 = 30$  nautical miles and then along a path of length  $3 \cdot 15 = 45$  nautical miles. The angle between the two paths is  $110^\circ$ . (You can find this angle by drawing north-south lines and using geometry.) To find  $x$ , the distance of the ship from its starting point, use the law of cosines:

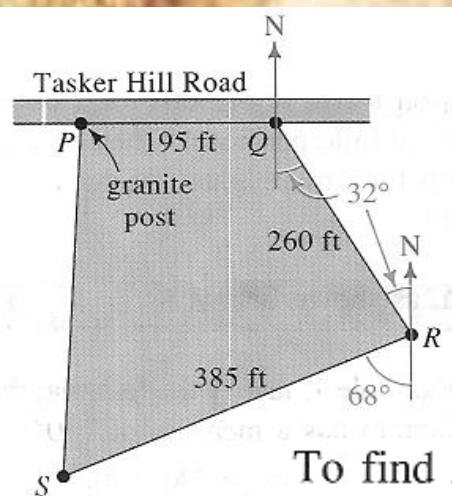
$$x^2 = 30^2 + 45^2 - 2 \cdot 30 \cdot 45 \cdot \cos 110^\circ \approx 3848$$

Thus,  $x \approx \sqrt{3848} \approx 62.0$  nautical miles.



Example 2. Very often a plot of land is taxed according to its area. Sketch the plot of land described. Then find its area.

From a granite post, proceed 195 ft east along Tasker Hill Road, then along a bearing of S32°E for 260 ft, then along a bearing of S68°W for 385 ft, and finally along a line back to the granite post.



From the bearings given, we deduce that:

$$\angle PQR = 90^\circ + 32^\circ = 122^\circ$$

$$\angle QRS = 180^\circ - (32^\circ + 68^\circ) = 80^\circ$$

To find the area of  $PQRS$ , we divide the quadrilateral into two triangles by introducing  $\overline{PR}$ . Area of  $\triangle PQR = \frac{1}{2} \cdot PQ \cdot QR \cdot \sin Q$

$$= \frac{1}{2} \cdot 195 \cdot 260 \cdot \sin 122^\circ \approx 21,500 \text{ ft}^2$$

To find  $PR$ , we use the law of cosines:

$$PR^2 = 195^2 + 260^2 - 2 \cdot 195 \cdot 260 \cdot \cos 122^\circ \approx 159,000 \quad PR \approx 399 \text{ ft.}$$

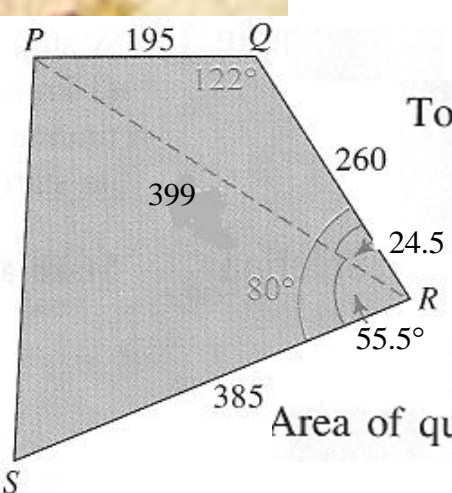
To find  $\angle PRS$ , we find  $\angle PRQ$  by the law of sines:

$$\frac{\sin PRQ}{195} = \frac{\sin 122^\circ}{399} \quad \sin PRQ = \frac{195 \sin 122^\circ}{399} \quad \angle PRQ \approx 24.5^\circ$$

Therefore,  $\angle PRS = \angle QRS - \angle PRQ \approx 80^\circ - 24.5^\circ = 55.5^\circ$ .

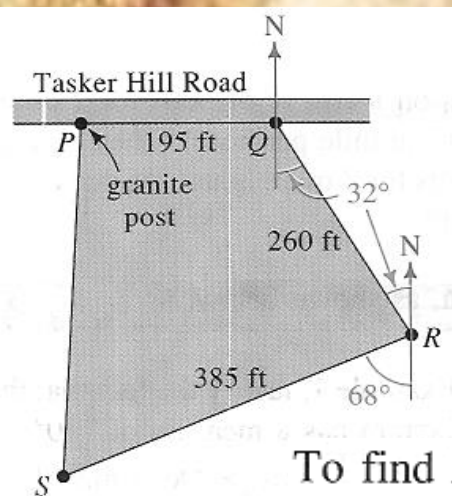
$$\text{Area of } \triangle PRS = \frac{1}{2} \cdot PR \cdot RS \cdot \sin PRS \approx \frac{1}{2} \cdot 399 \cdot 385 \cdot \sin 55.5^\circ \approx 63,300 \text{ ft}^2$$

$$\begin{aligned} \text{Area of quadrilateral } PQRS &= \text{area of } \triangle PQR + \text{area of } \triangle PRS \\ &\approx 21,500 + 63,300 \\ &= 84,800 \text{ ft}^2 \end{aligned}$$



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