Sec 9.5
Applications of Trigonometry to Navigation and Surveying
Which direction?

- In basic Trig... standard position:

  Start on the x-axis. Counter clockwise
Which direction?

- Navigation... used by ships, planes etc.

Start on the y-axis.
Clockwise
Given using 3 digits
9.5 Applications of Trigonometry to Navigation and Surveying

**Objective**

To use trigonometry to solve navigation and surveying problems.

The course of a ship or plane is the $\angle$, measured clockwise, from the north direction to the direction of the ship or plane.

The course of ship $= 110^\circ$
9.5 Applications of Trigonometry to Navigation and Surveying

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Bearing of $B$ from $A =$

Bearing of $A$ from $B =$

The course of a ship or plane is the angle, measured clockwise, from the north direction to the direction of the ship or plane.
Example 1. A ship proceeds on a course of 300º for 2 hours at a speed of 15 knots (1 knot = 1 nautical mile per hour). Then it changes course to 230º, continuing at 15 knots for 3 more hours. At that time, how far is the ship from its starting point?

Make a diagram
Always measure clockwise
A ship proceeds on a course of $300^\circ$ for 2 hours at a speed of 15 knots (1 knot = 1 nautical mile per hour). Then it changes course to $230^\circ$, continuing at 15 knots for 3 more hours. At that time, how far is the ship from the starting point?

Law of Cosines

\[ (OPP)^2 = (ADJ_1)^2 + (ADJ_2)^2 - 2(ADJ_1)(ADJ_2)\cos(\angle) \]
Law of Cosines

\[ (OPP)^2 = (ADJ_1)^2 + (ADJ_2)^2 - 2(ADJ_1)(ADJ_2)\cos(\angle) \]
Which direction?

- In surveying, a compass reading is usually given as an acute angle from the north-south line toward the east or west line.

a) Start on the y-axis.

b) Clockwise

c) The angle is always acute.
In surveying, a compass reading is usually given as an acute \( \angle \) from the north-south line toward the east or west.
Azimuths

<table>
<thead>
<tr>
<th>Azimuth</th>
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<tbody>
<tr>
<td>315°</td>
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<tr>
<td>270°</td>
</tr>
<tr>
<td>225°</td>
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<tr>
<td>180°</td>
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<tr>
<td>135°</td>
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<tr>
<td>90°</td>
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<tr>
<td>45°</td>
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</tbody>
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Bearings

<table>
<thead>
<tr>
<th>Bearing</th>
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<tbody>
<tr>
<td>N 45° W</td>
</tr>
<tr>
<td>N 45° E</td>
</tr>
<tr>
<td>W</td>
</tr>
<tr>
<td>S 45° W</td>
</tr>
<tr>
<td>S</td>
</tr>
<tr>
<td>S 45° E</td>
</tr>
</tbody>
</table>

Navigation

Surveying
NE Sandy and NE 40\textsuperscript{th} meet at approx 58 degree angle.

Give directions from the Formation Area to the disband Area.

a) Using navigation system.

b) Using the survey method.
Basic Hints and rules

- Make a diagram... give yourself drawing space *all* around the diagram.
- Although drawing to scale might be hard, come as close to a scale as possible.
- Write all the given information on your diagram.
Basic Hints and rules

• Include a lightly drawn x and y axes at each point.
• Find as many angles and sides as you can.
• Apply as many geometry rules as you can.
18 meters Reference

2 Degree difference
Camping:

Give direction from the camp site to each of 4 points of interest:

a) The river
b) The lake
c) To the tower
d) The hill
Example 2. Very often a plot of land is taxed according to its area. Sketch the plot of land described. Then find its area.

From a granite post, proceed 195 ft east along Tasker Hill Road, then along a bearing of S32°E for 260 ft, then along a bearing of S68°W for 385 ft, and finally along a line back to the granite post.

With these types of problems, a careful diagram is essential. Next slide will demonstrate all the steps. Make sure to have your geometric tools and math wits about you!
area of $\triangle GQS = 35,561.49$

area of $\triangle GRS = 49,289.63$

TOTAL: $84,851.12$
A plot of land is taxed according to its area. Sketch the plot of land described, then find its area. 

From a granite post, proceed 195 ft east along Tasker Hill Road, then along a bearing of S32°E for 260 ft, then along a bearing of S68°W for 385 ft, and finally along a line back to the granite post.

\[
k = \frac{1}{2} ab \sin C
\]
500 m NE;

15. **Triangulate**

\[ d \approx \]

\[ \sin \theta = \frac{200}{200} = \]

\[ m \angle \alpha = \]

\[ K_2 = \]

\[ d^2 = \]

**Law of Sines**

\[ \angle \theta = 15^\circ \]

\[ 90^\circ \]

\[ 300 \text{ m} \]

\[ 200 \text{ m} \]

\[ 500 \text{ m} \]

\[ 45^\circ \]

\[ K_1 = \]

\[ \text{Area} \approx \]
260 m SW; 240 m S; 280 m N 40° E; Back

\[ d \approx \]

\[ \frac{\sin \theta}{240} = \]

\[ K = \]

\[ d^2 = \]

\[ K = \]

\[ \text{Area} \approx \]
Proceed S78 W for 250 m.
Then S15 E for 180 m.
Then N78 E.
Then N30 E to the starting point.

\[
\frac{\sin \theta}{180} = \frac{\sin 87^\circ}{300.32}
\]

\[
(d_3)^2 = \frac{d_1}{\sin 11.2^\circ} = \frac{300.32}{\sin 132^\circ}
\]

\[
K_1 + K_2 \approx
\]
• **click here for a sample test**
Homework:

- Sec 9.5 written exercises
- 7-13 odds; 15, 16, 17

In class Exit Slip:

- Page 353 Problem #17.
- Only full solutions will be considered.

If you were absent, see Navi for make up Exit Slip.
Applications of Trig to Navigation and Surveying

The course of a ship or plane is the angle, measured clockwise, from the north direction to the direction of the ship or plane.

- Course of ship: $110^\circ$
- Course of plane: $270^\circ$
As shown, the compass bearing of one location from another is measured in the same way. Note that compass bearings and courses are given with three digits, such as 060° rather than 60°.

bearing of $B$ from $A = 060°$
bearing of $A$ from $B = 240°$
Example 1. A ship proceeds on a course of 300° for 2 hours at a speed of 15 knots (1 knot = 1 nautical mile per hour). Then it changes course to 230°, continuing at 15 knots for 3 more hours. At that time, how far is the ship from its starting point?
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Make a diagram

The ship travels first along a path of length 2 \cdot 15 = 30 nautical miles and then along a path of length 3 \cdot 15 = 45 nautical miles. The angle between the two paths is 110º. (You can find this angle by drawing north-south lines and using geometry.) To find \( x \), the distance of the ship from its starting point, use the law of cosines:

\[
x^2 = 30^2 + 45^2 - 2 \cdot 30 \cdot 45 \cdot \cos 110º \approx 3848
\]

Thus, \( x \approx \sqrt{3848} \approx 62.0 \) nautical miles.
Example 2. Very often a plot of land is taxed according to its area. Sketch the plot of land described. Then find its area.

From a granite post, proceed 195 ft east along Tasker Hill Road, then along a bearing of S32°E for 260 ft, then along a bearing of S68°W for 385 ft, and finally along a line back to the granite post.

From the bearings given, we deduce that:

\[ \angle PQR = 90^\circ + 32^\circ = 122^\circ \]
\[ \angle QRS = 180^\circ - (32^\circ + 68^\circ) = 80^\circ \]

To find the area of \( PQRS \), we divide the quadrilateral into two triangles by introducing \( PR \). Area of \( \triangle PQR = \frac{1}{2} \cdot PQ \cdot QR \cdot \sin Q \)

\[ = \frac{1}{2} \cdot 195 \cdot 260 \cdot \sin 122^\circ \approx 21,500 \text{ ft}^2 \]

To find \( PR \), we use the law of cosines:

\[ PR^2 = 195^2 + 260^2 - 2 \cdot 195 \cdot 260 \cdot \cos 122^\circ \approx 159,000 \]
\[ PR \approx 399 \text{ ft.} \]

To find \( \anglePRS \), we find \( \angle PRQ \) by the law of sines:

\[ \frac{\sin PRQ}{195} = \frac{\sin 122^\circ}{399} \]
\[ \sin PRQ = \frac{195 \sin 122^\circ}{399} \]
\[ \angle PRQ \approx 24.5^\circ \]

Therefore, \( \anglePRS = \angleQRS - \anglePRQ \approx 80^\circ - 24.5^\circ = 55.5^\circ \).

Area of \( \triangle PRS = \frac{1}{2} \cdot PR \cdot RS \cdot \sin PRS \approx \frac{1}{2} \cdot 399 \cdot 385 \cdot \sin 55.5^\circ \approx 63,300 \text{ ft}^2 \)

Area of quadrilateral \( PQRS = \text{area of } \triangle PQR + \text{area of } \triangle PRS \approx 21,500 + 63,300 \]
\[ = 84,800 \text{ ft}^2 \]
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Area of \( \triangle PRS \)

\[ \frac{1}{2} \cdot PR \cdot RS \cdot \sin PRS \approx \frac{1}{2} \cdot 399 \cdot 385 \cdot \sin 55.5° \approx 63,300 \text{ ft}^2 \]

Area of quadrilateral \( PQRS \) = area of \( \triangle PQR \) + area of \( \triangle PRS \)

\[ \approx 21,500 + 63,300 \]
\[ = 84,800 \text{ ft}^2 \]